Alejandro M. Rosas
Editor

Research reports in Mathematics Education: the classroom

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Foreword

To propose and develop an investigation is similar to a person´s journey over an imaginary city, there is a starting point, an initial question that places the researcher in a geographical place where it is possible to look in several directions.

One can look at theories as a kind of compass that target a route, one can make the decision to dive into the darkness to advance groping and try to overcome the obstacles that are found step by step, or one could even ask to other travelers how they planned their travel, what sites they visited, what roads were more interesting or not.

There are “The Research Manuals” that work as a travel guide which tell the tourist, where to start, what places to visit, what path to follow, and of course, how far to go. For someone that does not want to suffer the inconveniences of not knowing where to go, it is an essential option. However sometimes you want to discover things by yourself, even if this means more time and sometimes walking in circles without knowing it.

When you start this travel you know, more or less where to get. But actually it is not the final point that matters so much, but how was the route raised, which were the biggest obstacles, which paths were the most interesting, in summary, what experiences and reflections led to that route. It is clear that the interpretation of each place can be very different from person to person, interests, prior knowledge as well as the time you want to spend to study it.

This is how a person can give a very different explanation of the same fact, however this does not imply that it is wrong. There are simply aspects or features that were not considered or observed in the first instance, which is why it is sometimes necessary to revisit them. Upon completion of the investigation one may feel that this or that question was missing. And it is true, there are questions that are not exhausted or that, due to tack of certain background, it was not possible to observe them. Nevertheless, the aspiration of this book is to serve as a reference for other travelers, so they can see the route raised and the issues observed.

For me, CICATA is a place where these routes are projected and planned, and the expert view of the teachers, the compass that guides and warns of the problems that will be inevitable face in our journey, but from which we will get a lot of experience and learning. My recognition to all of them, for the humanity that characterizes them, for the expeditionary and inquiring spirit that floods the entire academic project in the ProME, for questioning again and again the theoretical hegemony of the false prophets who insist in selling you the right map for your travel, but above all, the commitment that characterizes them.

However, although there is someone that is always attentive to your journey, as Billie J. Armstrong pointed;

I walk a lonely road
The only one that I have ever known
Don't know where it goes
But it's only me, and I walk alone
I walk this empty street
(Green Day, 2004)

The road is lonely because in the end you decide where you are going. And that is the best of all, the independence and initiative of the students to take the paths that they want to walk. Something that deep inside, in the background every teacher wants from their students.

The travel logs (research reports) that make up this interesting book are articulated by the same concern, explore innovative scenarios for learning mathematics. This means expanding the horizons of current teaching to rethink teaching from seemingly distant places, so that the terms "proposal", "strategy", "scenario", "opportunities", "teaching sequence" are present in the titles of works with a provocative tone.

The first encounter with the readings detonated several reflections in me, which meet in one: originality. Not only for the novelty of the approaches, but for the courage to reflect on issues that usually go unnoticed in teaching, for example: What situations are required for the study of statistics in a specific professional scenario? How can you establish a link between arithmetic and algebra? Or what opportunities does geometry provide for the study of differential calculus?

Undoubtedly, the issue of riddle questions, the (traditional) school idea of reasoning in the face of a problem, or the case of projects that propose a way of organizing learning very far away from the traditional exposure – repetition model.

It is therefore an invitation to immerse yourself in reading these reports, where you can find valuable testimonies of the routes faced by brave expeditionaries and talented teachers with a single purpose: to know.

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Preface

The Mathematics Education Program (ProME for the Spanish Programa de Matemática Educativa) of Instituto Politécnico Nacional, located in Mexico City, began its activities in September 2000 with the offer of a Master's program in Science in Mathematics Education and a Doctorate program in Mathematics Education.

As part of the academic activities of ProME's community, research, research projects, didactic activities, didactic sequences, proposals for study programs, etc. have been generated. The publication of all that scientific production was the next natural step.

The editorial program of ProME was born in 2011 and since then 19 books have been published in 5 different editorial series. And from that moment each of the books is available for free download on ProME's website (https://www.cicata.ipn.mx/oferta-educativa/prome/programa-editorial.html).

In this year we started a new series of books written in English called *Research Reports in Mathematics Education*. And it has its first issue entitled *The Classroom* because the research articles contained in it address research developed in the classroom.

At ProME we hope that this editorial adventure in English will be long and fruitful for us, and that at the same time it will be intellectually stimulating and useful for the scientific community.

Alejandro Rosas

November 2019.

Rosas, A. (Ed.) (2019). *Research Reports in Mathematics Education: the classroom*. Miami, FL: L.D. Books.

Riddles: setting for the development of logical and mathematical thinking in high school education

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Abstract

This research is a qualitative study that aims at increasing the development of mathematical-logical thinking for second-grade students at junior high school education. The purpose is to solve riddles. The proposal arises from a ludic focus, which is based on the Danesi (2003) and De Guzmán (1989) contributions about the recreation of the work of mathematicians and the correspondence between game rules and mathematical thinking. Results show that the implementations of various strategies favor the development of mathematical thinking and the evolution of such strategies.

Key words: Riddle, Mathematical logical thinking, Junior High School Education.

Introduction

According to Gomez and Villegas (2007), high school teachers normally face problems in the classroom such as poor understanding of written texts, specifically, those that focus on the symbolic language. This is why, some considerations are being made, and these are based on the difficulty of logical thinking processes. The logical and mathematical thinking is directly linked to

Cortez, R., Cuevas, A., & Castillo, D. (2019). Riddles: setting for the development of logical and mathematical thinking in high school education. In A. Rosas (Ed.), *Research Reports in Mathematics Education: the classroom* (pp. 3-14). Miami, FL: L.D. Books.

the ability to work and to think in terms of numbers and the capacity to make use of the logical thinking: observation, imagination, intuition and logical thinking are skills that favor its development. In this sense, it is of paramount importance the necessity to implement strategies that empower the processes in logical thinking in the classroom. Furthermore, it was considered the use of riddles as an alternative for students' development based on the existing match between the game rules and the development of the mathematical thinking (Danesi, 2003; Gairín, 1990).

Background

There is some research and contributions oriented to highlight the close relationship between the game and the teaching of Mathematics, Winter and Ziegler, 1989, (cited by Gairing, 1990), state schematically the match between the rules of the games and the mathematical thinking:

Table 1. *Correspondence between the rules of the games and mathematical thinking.*

Games	**Mathematical Thinking**
Rules of the games	Rules of construction, logical rules and operations.
Initial situations	Axioms, definitions.
Games	Constructions, deductions.
Figure of the game	Means, expressions, terms.
Game strategy	Skillful utilization of the rules of reduction in known exercises known to rules.
Resulting situations	New theorems, new knowledge.

Given the similarity of structure between the game and mathematics, which is observed in the former chart, it is clear that there are some activities and attitudes in the mathematical thinking which is observed in the search of solutions of games. This occurs, for example, when using heuristic techniques, communication and observation abilities, as well as, by appreciating the power and beauty of the mathematical argumentation. Furthermore, such activities and attitudes allow students to reinforce and develop mathematical knowledge. For example, this is evident in the magical charts and the number guessing where the use of Arithmetic is present, particularly, in the use of riddles resolution of ages and measurements where the Algebra is also used. Another example is when students have to solve puzzle dissections where the use of Geometry and Logics is used, specifically, in the use of riddles and paradoxes, among others.

In 1989, De Guzman argued that a well-chosen game may guide students to the best vantage point of observation and initial approximation, and therefore, this may be useful for students to face any study challenge. As for Batllori (2013), propose the analysis of a game and to search for solutions as an activity similar to the work of mathematicians when these provide the ideal setting for finding mistakes and learn from these in the practice. Dieudonné (cited por Gairín, 1990) points out that the mathematical tasks, the mathematical exercises always show a double origin: on one hand, there are problems that come from technical problems, on the other hand, there are problems that come from the curiosity, that is to say, riddles.

According to Danesi (2003) and Fernández, León and García (2017), riddles are playful instruments, such as a text and object that has been designed to provide distraction and empower skills, moreover, the characteristics of these texts are closed –meaning that they conduct to one solution- and open –which are solvable- and they never show a problem either in obvious or evident form.

Riddles have been of great interest for mathematicians (Tartaglia, Cardano, Fibonacci, among others), and those who are not mathematicians. In this sense, there are some research and contributions that stand out the potentiality of riddles in the teaching-learning of mathematics. Tuapanta (2014) reports that the utilization of recreational mathematics while working and by using problem solving sentences (riddles) significantly increased the development of logic mathematical reasoning in students at first term in high education level. Malaspina (2016) had a didactic experience on a riddle, and found that by analyzing, guessing, demonstrating and creating are forms for stimulating the mathematical thinking. Vanegas, Henao and Gustin (2013) agree that crossing riddles and figures of only one stroke may be used in the classroom; this is in order to encouraging spatial vision, abstraction problems, deduction and generalization capacity.

As for Murray-Lasso (1999), authors observe that by using riddles is possible to introduce concepts, solution methods, and practical applications in the teaching of Operation Research. Likewise, Núñez, Pérez, Bueno, Diánez and Elías (2004) developed a proposal in order to introduce high school students in the study of the Combinatory and the Introduction in the Graph Theory, from the riddles of *bridges of Königsberg.* Similarly, Velasco in 2017 examined various solutions which were given by talented students in high school in Mexico. These students used the *snail riddle.* The author reports that the resolution of riddles may be a good predictor of students' performance in the physics final examinations. On the other hand, Fernández, León and García (2017) recognize the use of mathematical riddles as educational instruments: the authors further enlist contents that can be used in higher education programs (Theory of Numbers, Combinatory, Descriptive Statistics, Functions, Equations, Inequations, Systems of Equations, Plain Geometry, Analytical Geometry and Geometry of Space).

Methodology

This is a qualitative research which shows a non-probabilistic sample of 54 students at second grade at the Juan Espinosa Bavara High School, in morning shift. Riddles were chosen as a text or playful object that has been designed for distraction and for empowering skills. This research went through the following stages:

Stage I. Determination of riddles and analysis categories.

The selection of riddles was made by following two criteria: contents in high school programs and contribution to skill development that favor the logical and mathematical thinking. It was determined to implement 20 riddles that correspond to the categories proposed by Batllori (2013): memory, strategy, puzzles, mental agility, and mathematical calculation.

As for the unit analysis, this was mainly focused on:

- Use of concepts of mathematics notions.
- Procedural skills.
- Strokes and diagrams.
- Perception of properties and relationship among themselves.
- Alternative exploration.

Stage II. Implementation and analysis of results.

Riddles were taught in class as a systematic activity that promoted knowledge. There was a 7-10 minutes introductory period of time through which instructions were given. In addition, students were given feedback to their possible questions or doubts, and finally, there was a collective closing of the lesson. Thus, by taking into account the students' production, it was identified and set into categories as follows: *the use of concepts of mathematics notions and the strokes and diagrams, where riddles were organized into from points to pairs and "ladies and bargeman".*

The first of them was implemented with the purpose of developing imagination, which is enriched through activities that allow an array of alternatives (Gómez y Villegas, 2007).

This riddle points out that:

"Match these four points with 3 straight lines by following one condition: one should stick the pencil to the paper at all times in order to draw the lines. Also, one should not write the line twice in the same direction to the point".

In this task, strategies that are more recurrent are shown in figures 1, 2, and 3.

Figure 1. Strategy in line strokes with intersection.

Cortez, R., Cuevas, A., & Castillo, D. (2019). Riddles: setting for the development of logical and mathematical thinking in high school education. In A. Rosas (Ed.), *Research Reports in Mathematics Education: the classroom* (pp. 3-14). Miami, FL: L.D. Books.

Figure 2. Strategy stroke of three consecutive straight lines.

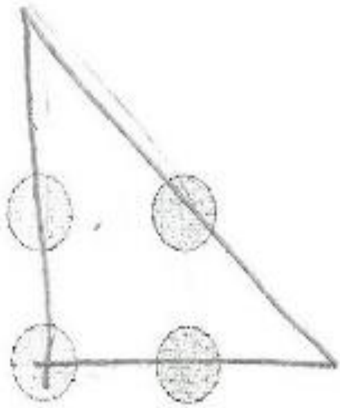

Figure 3. Strategy of strokes of three consecutive lines that are united.

In figures 1 and 2, it is observed that students drew three lines without writing twice in the same direction. This was possible by planning intuitive processes, however, in figure 3, it is clear that the logical reasoning was employed by uniting all points; and, by considering that uniting means to put together two or more elements in order to form a unit.

The second riddle was used to reaffirm basic concepts of Combinatory which can be given through crossing riddles (Gairín, 1999 y De Guzmán, 1989). This riddle establishes that:

"Many years ago, three ladies coming from a far castle were traveling with three servants when they approached the banks of a river, which they had to swim across. There was no bridge but a little boat that could fit two people only.

The ladies did not trust in their servants because they thought one of them wanted to kill one of the ladies. Given this gruesome reason, they agreed not to stay apart and be alone with two servants or two of them (ladies) with three servants.

Following this norm: How were these ladies able to cross the river without any irreparable misfortune?"

In this exercise many students worked out their proposals of solution (see: figures 4, 5, and 6); some of the schemes were iconic as it can be seen in figure 4.

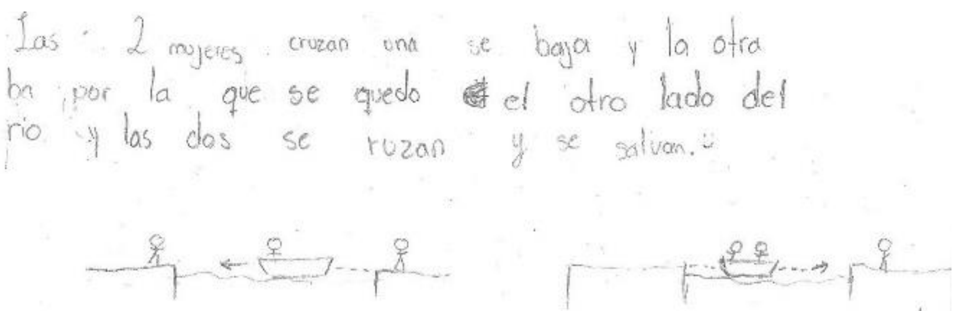

Figure 4. Resolution using iconic scheme

In some other cases such in figure 5, it is observed one scheme that introduces literals as a support for natural language and the use of one strategy that is already known.

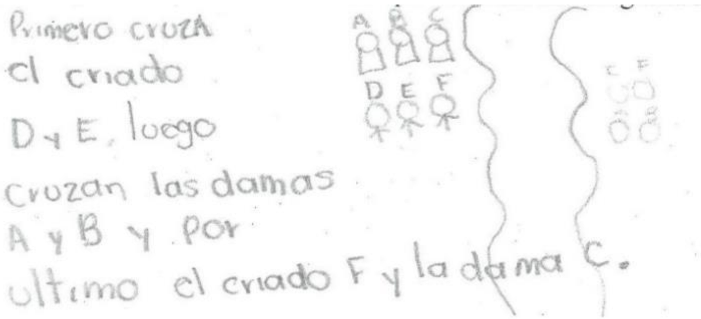

Figure 5. Resolution supported by literals

From figures 4 and 5 it is fair to say that the student did no put into practice the notions of Combinatory.

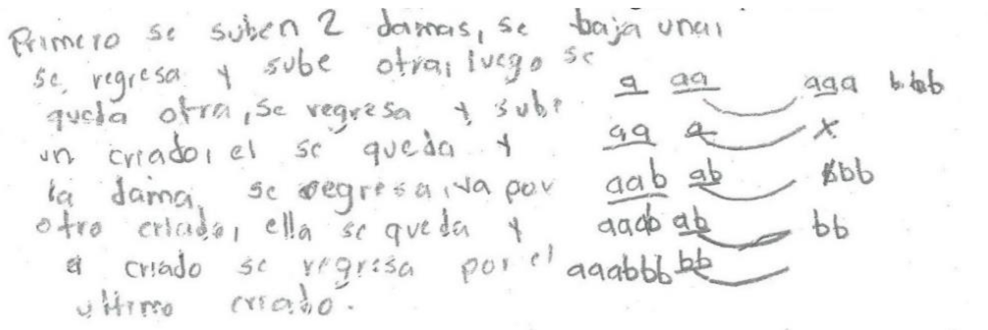

Figure 6. Resolution expressed in symbolic language.

In figure 6, the scheme employs a symbolic language. It also suggests that the strategy used is already known. Likewise, it reveals that the notion of mathematics concept.

Discussion and conclusion

The introduction of riddles in the classroom set a scenery for the development of the logical-mathematics thinking. This was possible because students put into practice skills: observation, deduction, experimentation and argumentation. With this, it was confirmed what Malaspina (2016) y Vanegas, Henao y Gustin (2013) reported about the stimulation for the mathematical thinking applied from the riddles.

The crossing riddle allowed us to smoothly work on some aspects of the Combinatory. This was used for second grade students at high school. This activity coincides with what Núñez, Pérez, Bueno, Diánez and Elías (2004) reported as they utilized the riddles in *forms to carry out tasks or crossing riddles* in the introduction of combinatory in a simple way.

In sum, if we use as reference the plurality of alternatives used by students, it is possible to notice the differences in the treatment of logical-mathematical thinking as well as the noticeable motivation in the Mathematics lessons that provides certainty to what was pointed by Fernández, León and García (2017), the mathematical riddles are a motivational resource

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Personalized Mathematical Exercises as a teaching strategy to reduce copying in a high school math class

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Abstract

The experiences lived in a math class are very diverse, for example, some relate to the teacher's way of teaching, others to the student's way of learning, to the problems in the teaching-learning process. For this research work, attention was focused on one of the problems that arise in the teaching-learning process of mathematics that is often left aside but that is very recurrent among students, it is known as "cheating". This school event led to the implementation of what is described in this work as the Personalized Mathematical Exercises (EMP) model; The reason why it was decided to focus research on this model in a math class at the high school level is because it is considered that with this teaching strategy the practice of copying decreases considerably.

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Key Words: Mathematics learning; cheating; copy

1. Literature Review

The research carried out around the problem of "copying" in a math class that was consulted for this work was varied and some have individual, others social, behavioral, statistical approaches.

Rosas, A., Cuenca-Villamonte, S., & Molina, J.G. (2019). Personalized Mathematical Exercises as a teaching strategy to reduce copying in a high school math class. In A. Rosas (Ed.), *Research Reports in Mathematics Education: the classroom* (pp. 15-40). Miami, FL: L.D. Books.

In Šorgo, Vavdi, Cigler and Kralj (2015) there is an investigation conducted in Slovenia, the authors found that cheating is a common practice in schools in that country, and that almost all students do it. These dishonest actions occur more in homework, but also in exams. There are questions that arise in view of professional experience, such as why cheat in school? This and other questions were asked by Simmons (2018) in his investigation. As a result, it establishes 5 strategies to reduce cheating. In the same vein Elliot, Deal and Hendryx (2014) had found that in an art school 87% of students had cheated. Although they found that the students least likely to cheat are those students who believe that the punishment for cheating should be stricter. This despite the fact that one reason to cheat is to help a partner.

In Chudzicka-Czupała, et. al (2016) the reasoned action theory, the planned behavior theory and the modified and extended version of the latter in several countries, in relation to academic dishonesty, were evaluated and compared. They are explained in more detail in the methodology section of this document. The author wanted to evaluate and compare how these theories predict the predisposition of some students to copy. The predictive capacity of the reasoned action theory, the planned behavior theory and the four-factor model of the planned behavior theory (moral obligation model) on the intention to engage in dishonest academic behaviors was compared. It was assessed whether the individual elements of the Ajzen model (attitudes, subjective social norms, perceived behavioral control and moral obligation) explain the intention to participate in academic dishonesty, and whether these key predictors vary across cultural configurations. It was found that the modified theory of planned behavior emerged as the best and most complete basis for understanding the history of fraudulent behavior, highlighting the factors that influence students' intentions to deceive and identify dominant factors. The universal character of the theory, which includes elements present in all cultures (attitudes, subjective social norms, perceived behavioral control and moral obligation), makes it possible to make comparisons between students of different nationalities and offers a solid framework from which to establish and align the expectations in cohorts of culturally diverse students.

Anderman and Won (2017) investigates the beliefs of cheating in courses that displease; finding that students consider and believe cheating in math and science courses is acceptable. In McCabe D.L., Trevino, L.K., and Butterfield, K. D. (2001), a decade of research on copying in academic institutions was reviewed. The ethical inclinations of business students were analyzed. The objective of this research focused on one of the most basic ethical decisions that university students face: cheating or not cheating in their academic work. With the growing competition for the most desired positions in the labor market and for the few coveted places available in the main business, law and medicine faculties of the country, undergraduate students experience considerable pressure to obtain good results. The authors wanted to determine the frequency of academic pitfalls and the factors that can have a positive or negative influence to perform them. They surveyed more than 5,000 students in a diverse sample of 99 colleges and universities in the United States and discovered that three-fourths of the respondents had been involved in one or more incidents of academic dishonesty. This study was replicated 30 years later by McCabe and Trevino in 1997, in 9 of the schools that had participated in Bowers. Although these studies made important contributions, most of them had significant limitations. Perhaps of the greatest importance, most of these studies only sampled students in a single institution, which obviously limits our ability to draw meaningful conclusions about contextual influences.

It was shown that traps are frequent and that some forms of trap have increased dramatically in recent years. Suggestions could be established to handle the traps from the perspectives of students and teachers, for example, the codes of honor in academic integrity programs and policies of an institution can have a significant influence on student behavior. Research shows that all too often, pressures to obtain good positions lead to decisions to participate in various forms of academic dishonesty. Research also shows that these transgressions are often overlooked or treated lightly by professors who do not want to get involved in what they perceive as bureaucratic procedures designed to adjudicate accusations of academic dishonesty on their campus. Students who could otherwise complete their work honestly observe this phenomenon and convince themselves that they cannot afford to be disadvantaged by students who cheat and are not denounced or punished. One of the most encouraging aspects of research is the number of students and professors who are really concerned about academic dishonesty and who are willing to devote time and effort to address it on their campuses. It was revealed that factors such as gender, grade point average, work ethic, behavior, competitive effort and self-esteem can significantly influence the prevalence of the trap.

Nonis and Swift (2001) sampled business students (from all business disciplines) on six different campuses to find factors that foster academic dishonesty for what they sought to make dishonest behavior measurements in university classes of the students. From this, the authors obtained two main results: (a) Students who believed that dishonest acts are acceptable were more likely to participate in those dishonest acts than those who believed that dishonest acts were unacceptable, and (b) students who they participated in dishonest behaviors in their university classes were more likely to participate in dishonest behaviors at work. The results suggest that if students do not respect the climate of academic integrity while in college, they will not respect integrity in their future professional and personal relationships. Education and communication can create a shared commitment to academic integrity among students, faculty and administrators. It is essential that institutions demonstrate a commitment to the implementation of academic dishonesty policies and provide the resources to help avoid classroom traps. Students have indicated that, when they feel like real members of the campus community, they believe that faculty members are committed to ethical standards and know the policies of their institutions regarding academic dishonesty. When students support the standards of academic integrity, they realize their responsibilities regarding ethical behavior. This core value of the institution becomes its own individual core value, which lead to future careers.

Pavlin-Bernardić, Roban, and Pavlović (2016) investigated the frequency of cheating of high school students in Mathematics and the relationships between deception and motivational beliefs. Two different types of traps were examined: 1. Active, which aims to increase people's own success and third-party deception, aimed at helping other students achieve success. Students use third-party traps very often and more than active traps. 2.-Motivational, where beliefs are significantly related to active deception, but are not correlated with third party deception. Therefore, although active and third-party traps are classified as dishonest acts, they do not have the same motivational mechanisms at their bottom.

The conclusions they reached indicate that although active and third-party traps are considered to be dishonest acts, they do not have the same motivational mechanisms in their background. One of the contributions of this study is that it is one of the first studies to simultaneously examine student objectives, expectations of success and the perceived cost of cheating. Despite some limitations, the results of this study have practical implications for educators. Cheating academics is related to students' motivation to learn and their attitudes about cheating. Therefore, to reduce the pitfalls, teachers should try to arouse student interest in their subject and emphasize the usefulness of what is being learned. They should also try to link the course material, as much as possible, to the students' daily life. Teachers should also encourage mastery goals, creating a classroom climate with an emphasis on learning rather than at school.

Degrees or avoid work. As a suggestion, special attention should be given to talking with students about ethics and the unacceptability of cheating, regardless of context, and of building a school environment that encourages academic honesty. In Murdock and Anderman [9] students were asked three questions “What is my goal?”, “Can I do this?”, And “What are the costs?” The conclusions they reached were that Many of the individual and contextual factors that are related to cheating can be subsumed under a motivational framework. Some reasons mentioned were that low self-efficacy or high needs for achievement by being dishonest.

2. Problem Statement

Copy and plagiarism are very common and recurring practices in students of any subject and educational level, this has been reported in different research papers related to academic honesty, see for example, Pavlin-Bernardić, Roban and Pavlović (2016).

Mathematizing this dishonest practice to accurately determine the number of students you copy is very complicated, but why? From experience we know that copy and plagiarism have always been present in education; that is to say, perhaps as students we sometimes resorted to that practice or we observed that our classmates practiced it; even important politicians have been pointed out for it in Mexico and in other parts of the world.

There are many problems that exist in education, and the practice of copying is one of the most serious and to which, in addition, sufficient research time has not been dedicated to propose realistic and professional solutions that can be applied in classes of ordinary, real math, not in controlled environments.

PERSONALIZED MATHEMATICAL EXERCISES

The problem presented in previous sections led to the approach of a teaching strategy that I have called Personalized Mathematical Exercises (EMP), implemented in homework in class, at home and even in exams.

What do EMPs consist of?

All the exercises are structured in such a way that in the writing of the problem, the variable L is included in the data, so that the EMP of each student can be generated efficiently.

• Variable L is a number that will be assigned to the student as their enrollment or date of birth; To avoid conceptual complications and facilitate easy numerical comprehension in students, the variable L is handled with students as the list number of each of them, that is, a number belonging to the set of natural numbers or positive integers (set N) in the range of [1, + ∞). Students are high school level and according to the Mexican education system, they have been using natural numbers since primary school

In addition, it should be reviewed that with each list number there is a logical solution according to the topic so that unforeseen events do not occur, for example, responses related to negative roots or fractions with zero denominator (cases that do not have to be addressed with students of this level according to the study program).

• This number is then substituted by the student in variable L of the writing of the exercise so that the student knows their individual data, and that they differ with the data of their classmates.

• The teacher, to qualify or guide the students in these exercises, has the software or macros of the personalized mathematical exercises, where he types the student's list number so that the program yields the results for that case. The teacher can have programs or spreadsheets in Excel where each file corresponds to a class topic and in which each page of the file is a different exercise.

Below is an example of the writing and resolution of an EMP for the subject of geometry and trigonometry belonging to the second semester in a high school in Mexico:

EMP.- Obtain the perimeter of a rectangular terrain if the base is “2L” units and the height is “L + 2” units.

Note that, since L is a variable quantity that depends on a number assigned to the student (for this example it will be his list number), his data and answers will be different from his classmates.

Let's put as an example:

If a student is L = 1, his base will be 2, the height will be 3 and his perimeter will be 10.

Meanwhile, for the student with L = 20, their base will be 40, the height will be 22 and their perimeter will be 124 (see figure 2).

RESEARCH QUESTIONS

To combat the problem of copying and plagiarism in the different subjects and educational levels, the research explained in our literature review suggests that we implement humanistic strategies so that the student becomes aware that these practices are not correct; However, in this thesis we have chosen a strategy related to the structure of the tasks or exercises in math classes at the baccalaureate level, where the contributions of this strategy to the practice of copying will be analyzed

Now, what happens to the practice of copying in class if each student in a group in a math class is assigned a personalized math exercise? Would this practice decrease among students in this group? What complications would a teacher have when using personalized mathematical exercises in an ordinary class with approximately 30 students? What effect do personalized math exercises have on students who participate in the study in terms of the school assessments to which they are subjected? The above questions lead us to ask what contributions does the EMP model have as a teaching strategy in the face of the practice of copying in a math class at the baccalaureate level?

Hypothesis: The practice of copying in a high school math class will decrease with the implementation of the EMP model.

DISHONESTY AND ACADEMIC INTEGRITY

ACADEMIC DISHONESTY

Academic dishonesty is a very common problem in educational institutions and can be present at any educational level. If academic honesty is the virtue that an individual has to be sincere, decent, reasonable and fair in everything that refers to their education, academic dishonesty is the lack of that virtue. Academic dishonesty can come in the form of plagiarism, traps, copies, accordions, lies, use of unauthorized materials.

According to Chudzicka-Czupała et al. (2016), dishonesty or academic trap is any intentional action or behavior that violates the established rules that govern the administration of an exam or the completion of an assignment. It also gives a student an unfair advantage over other students in an exam or a task; or decreases the accuracy of intentional inferences that arise from a student's performance in a test or a task.

It is important to keep in mind that the social and cultural context of the student influences the way they see the academic trap, for example, a cheater student can be seen by some as an ingenious and brave young man while others as a lazy and corrupt young man.

ACADEMIC INTEGRITY

Each country has its constitution, each school has its institutional regulations and each teacher has its rules that must be respected in its class. Each group of individuals must be governed by rules that mark what is right and wrong so that everyone lives in a harmonious environment. Academic integrity can be understood as the ability of a person to act in accordance with the rules that are governed by an educational institution. Harp and Taeitz (1966), mention that without academic integrity standards, the stability and continuity of the academic system could not be maintained. It is well known that any violation of academic integrity standards is morally incorrect. Just as there must be rules of academic integrity, it is important that they be accompanied by sanctions in accordance with the non-compliance of said norms with the purpose not to punish, but to correct the person who failed to comply.

THEORIES RELATED TO THE ACADEMIC TRAP

THEORY OF REASONED ACTION

Theory of Reasoned Action (TRA) is a general model of the relationships between attitudes, convictions, social pressure, intentions and behavior. It was developed by Martin Fishbein and Icek Ajzen (in 1975-1980). Actions are based on individual attitudes, so a theory of action consists essentially of a description of attitudes. The information that allows their formation is cognitive, affective and behavioral. Cognitive information refers to the beliefs and knowledge we have about an object. Similarly, the information referred to other people is based on these components and is an important cause of the formation of our affective response. Behavioral information also influences attitudes, since we evaluate our own attitudes in a similar way to how we do it with those of others.

First, the measures of attitudes and behaviors must be compatible. That is, if the measure of attitude values a general attitude (towards an object, person or subject), then the measure of behavior must also be general. In contrast, if the attitude measure evaluates a specific attitude (towards a behavior), then the measure of the behavior must also be specific. The second factor that influences the congruence between attitudes and behaviors is the nature of the latter. Attitudes predict behaviors only when they are under the control of the will. The third factor is the nature of the attitude. Attitudes that are based on direct experience predict behavior better than attitudes based on indirect experience. The fourth factor influencing congruence between attitudes and behaviors is the personality dimension of self-supervision, which is an attribute that refers to the degree to which we rely on internal signals of behavior or external ones. Low self-supervision is based on relevant internal states, such as attitudes, values and beliefs, manifesting substantial congruence between attitudes and behaviors.

THEORY OF PLANNING BEHAVIOR (TCP)

The theory of planned behavior was developed in 1985, based on the Theory of Reasoned Action. This TCP theory contains five variables that include behavior, intention, attitude, subjective norm and control of perceived behavior. Unlike the theory of reasoned action, to the theory of planned behavior is added the control of perceived behavior, which refers to the perceptions of a person about the presence or absence of resources and opportunities required, however, this element it is not presented in the theory of reasoned action. Likewise, the planned behavior theory has proven to be superior to the reasoned action theory for predicting behavior. According to the theory of planned behavior, intentions and behaviors are a function of three basic determinants, one of a personal nature, another that reflects social influence, and the last one that deals with control issues.

According to Ajzen (1991), there are three types of beliefs related to the constructs of planned behavioral theory that are:

• Attitude: Behavioral beliefs - Beliefs they have about the likely consequences or other attributes of behavior.

• Subjective Standard: Normative beliefs - They are related to the normative expectations of other people.

• Perceived Behavior Control: Control beliefs - they have to do with the presence of factors that can hinder the performance of the behavior.

TEACHING STRATEGIES

According to Díaz (2009), teaching strategies are means or resources to provide pedagogical help adjusted to the progress needs of the students' constructive activity. To establish the distinction with respect to learning strategies, we appeal to the differentiation that cognitive psychology has been making for several years and is based on determining who is the main originator of the strategic activity.

If it is the student, these will be called “learning strategies” because they serve the student's own self-generated learning; if instead it is the teacher, they will be designated “teaching strategies” which also make sense only if they serve to improve student learning, although this sense is no longer self-generated, but encouraged, promoted or oriented as a result of the activity joint between teacher and students.

Here are some criteria as possible elements to consider for the selection and use of teaching strategies:

1.- Insert the activities carried out by the students, within a wider context and objectives where they make sense.

2.- Promote the participation and involvement of students in the various activities and tasks.

3.- Carry out, whenever possible, adjustments and modifications in the broader programming (of topics, units) and on the fly, always based on the observation of the level of action that students demonstrate in the management of tasks and / or the contents to learn.

4.-Make an explicit and clear use of language, with the intention of promoting the necessary intersubjectivity situation (between teachers and students), as well as the sharing and negotiation of meanings in the expected sense, trying to avoid ruptures and misunderstandings in teaching.

3. Methodology

The methodology for this investigation consists in the application of two instruments and in the explanation of the use of each one of them to achieve the objective of the investigation.

These instruments were designed to collect information related to EMPs, below are presented:

• Sheet composed of 4 exercises (2 traditional mathematical exercises and 2 personalized mathematical exercises).

• EMP test.

3.1 COMPOSITE LEAF OF 4 EXERCISES

The mathematical theme that is addressed with the implementation of the instrument is: surface of the circle. The reason for choosing this topic is because in the schedule of the classes, the topic should be seen on the dates on which it was planned to collect the data for the present investigation. The theme “surface of the circle” corresponds to the competence unit 4. Circumference and circle of the second part of the subject of geometry and trigonometry taught in the second semester of the baccalaureate level in Mexico for the general baccalaureate subsystem.

3.2 CHARACTERISTICS OF STUDENTS PARTICIPATING IN THE STUDY

The number of participating students was 29 where 17 were women and 12 were men. The average age of the students was 16 years. The students were enrolled in the second semester. The school institution is located in the State of Campeche, Mexico. The subject studied by students in which the implementation of the instrument was developed was geometry and trigonometry. The students, for the resolution of the worksheet, had already studied in previous classes concepts of arithmetic, algebra and geometric topics related to angles, triangles, quadrilaterals and circumference / circle.

CHARACTERISTICS OF THE 4 EXERCISES

The first two exercises have a traditional wording to the mathematical exercises of a standard class so we will refer to them from now on as Traditional Mathematical Exercises or equal for all (EMT). The last two exercises are Personalized Mathematical Exercises. The first and third exercise, as well as the second and fourth exercise, are actually similar exercises in relation to the same level of difficulty, same variables provided, variables requested and same procedures to be performed. The interesting thing about the experiment is the differences and similarities that are observed in the resolution of each student for the first and third exercise, or where appropriate, of the second and fourth exercise.

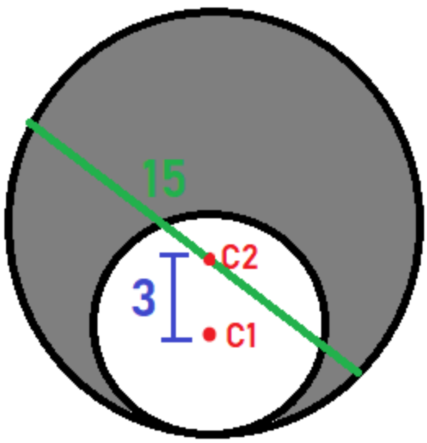
Below is the writing, procedure and solution of each of the 4 exercises that includes the sheet that students must solve in the session:

1.-Determine the area of a circle with a diameter of 160 units (leave the answer represented in two ways: in terms of pi and without pi).

A = Pi (160) (160) / 4 =

A = 6400 pi square units and A = 20106 square units.

2.-Find the value of the shaded area:



Distance between centers: 3 units

Large circle diameter: 15 units

Large circle area: Pi (15) (15) / 4 = 176.72 square units

Small circle diameter: ((15/2) -3) (2) = 9 units

Small circle area: Pi (9) (9) / 4 = 63.62 square units

Shaded area (subtraction from previous areas): 176.72-63.62 = 113.10 square units.

3.- Determine the area of a circle with diameter 10L units (Leave the answer represented in two ways: in terms of pi and without pi)

The data and the answers are based on the student's list number "L".

4.- Find the value of the area between the two circumferences of exercise 2.

3.3 A Priori Analysis

The objective of the implementation of the sheet composed of the 4 exercises is to observe the effect of the EMP in contrast to the EMT and to analyze if within the contributions we have the decrease of the practice of copying in a mathematics class of the baccalaureate level. In class the lessons learned will be evaluated; The teacher will write on the board 4 exercises to the students that must be delivered in loose sheets at the end of the session with procedure and result for each exercise (this instruction will be provided to the students). The teacher will leave class to not be a factor in the implementation that determines the behavior of the students; later the teacher will return to collect the worksheets. Due to the uncertainty that a student's copying attitude may be, it is based on the assumption that of the 29 participating students, there will be some who will resort to the practice of copying while the teacher is not in the classroom.

In the implementation, the three variables or beliefs of Ajzen (1991) related to the theory of planned behavior included in the theoretical framework are put into practice:

1.- The attitude or belief of the behavior.- It is based on the belief that many students with problems of dishonesty and academic integrity will resort to the practice of copying in the first two exercises that have the characteristic of being equal traditional mathematical exercises for all , that is, with equal data and answers for all.

Other students according to their attitudes they take may not.

2.- The subjective norm.- Classmates will positively or negatively affect each student in their attempt to practice the copy.

3.- The control of the perceived behavior.- It has to do with the presence of factors that can hinder the performance of the behavior, for example, the fact that the teacher is not at the time of performing the exercises influences because the students have the freedom to choose whether or not they copy according to their beliefs.

3.4 Mechanism for Obtaining Results

The mechanism used to find the number of students who copied in the implementation of the sheet of 4 exercises and that gives us the contributions of the EMP model is described below:

1.- The teacher qualifies the sheet of 4 exercises of each student through the following form:

The teacher assigns as grade for each exercise only one of the following letters: C, I, N; Below is the designation for each letter:

- C if the student's result is correct and included procedure.
- I if the student's result is incorrect and included procedure.
- N if the student did not perform a procedure, did not arrive at any response or did not do the exercise.

The teacher can use the EMP software to know if the student's result is correct and thus, to qualify in a more agile way.

2.-A data table is made where EMT is a Traditional Mathematical Exercise and EMP is a Personalized Mathematical Exercise.

3.- The four columns corresponding to results of exercises # 1 (EMT), # 2 (EMT), # 3 (EMP) and # 4 (EMT) will be filled with the letters (C, I, N) described in step 1.

4.- The last column of the table will be filled as follows:

- If the four columns corresponding to results of exercises have C, the opinion is: “No copy”.
- If the column in exercise 1 with exercise 3 and the column in exercise 2 with exercise 4 have the same letter (C, I or N), the opinion will also be “Not copied”.
- If in the column of exercise 1 he obtained C and in that of exercise 3 he obtained I or N, the opinion will be: “Copied”.
- If in the column of exercise 2 he obtained C and in that of exercise 4 he obtained I or N, the opinion will be: “Copied”.
- Any other unforeseen case will have an opinion of: “Indefinite”.

5.- The number of students who copied in the EMT but could not be copied in the EMP is equal to the sum of the cells with an “Copied” opinion.

In other words, this result is the number of students who, thanks to the EMP model, could not be copied in the two EMPs performed.

As an important note, to confirm the result obtained through the mechanism used and, if possible, obtain a more accurate result, the procedures of some students who obtained a “Copied” opinion will be observed to see the similarities and differences in their EMT and their EMP.

3.5 EMP TEST

The number of participating students was 207. The average age of the students was 16 years. Students were enrolled in the first semester of High School. The course in which the instrument was implemented was basic algebra. The students to answer the EMP test already had 6 months working with this type of exercises in their basic algebra classes, in other words, they are students who were already finishing their first semester of basic algebra.

3.6 CHARACTERISTICS OF THE TEST

The test consisted of 7 questions (2 multiple-choice questions and 5 open-ended questions). The objective of the test questions was to have real information related to the disadvantages and advantages that students noticed with the use of the EMP model, since they participated a whole semester in their implementation in class. The test was carried out on a Google® form (Google Form) and uploaded to the Classroom® platform of the classes so that the students will answer it within 24 hours (see figure 9). The test includes questions such as: How often have you copied in math classes (middle and high school)? Why do you think there are math students that copy? What effects did you observe in your academic training in mathematics with the Personalized Mathematical Exercises?

4 RESULTS

4.1 FILLING THE TABLE

The filling of the table was as follows:

Table 1. Four examples of how the table was filled.

L	Student	#1 (EMT)	#2 (EMT)	#3 (EMP)	#4 (EMP)	Result
1	Paulina	C	C	I	C	Copy
15	Darani	C	C	C	C	No copy
33	Giovanna	C	I	C	I	No copy
38	Mariana	C	C	I	I	Copy

Performing the count, we observed that 16 students (approximate amount obtained through this mechanism) resorted to the practice of copying in the first two traditional mathematical exercises the same for all.

4.2 POSTERIORI ANALYSIS OF PROCEDURES OF SOME STUDENTS WITH THE OPINION OF "COPY"

To confirm the results, a thorough analysis of the procedures of the exercises of 4 students who obtained an “Copied” opinion was carried out. The reason for having selected these 4 students was because of the obvious way in which they obtained their “Copió” opinion and that they can also be divided into these two cases:

Case 1_ They obtained correct results in exercises 1 and 2 while in exercises 3 and / or 4 they obtained “N” results, that is, they did not do the exercises.

Case 2_ They obtained correct results in exercises 1 and 2 while in exercises 3 and 4 they obtained incorrect results.

Below is the analysis performed on the worksheet of each of these 4 students by dividing them into cases 1 and 2 mentioned above.

Case 1_ They obtained correct results in exercises 1 and 2 while in exercises 3 and / or 4 they obtained “N” results, that is, they did not do the exercises.

Student with L = 2

The student performed the procedure and obtained the correct answer in exercise 2 while in 4, which was a similar exercise but with the characteristic of being EMP, not even tried to do so. This case gives indications that the student was copied in exercise 2.

Student L = 8.

We observe that the student performed the procedure and obtained the correct answer in exercise 1 and 2 while in exercises 3 and 4, which were similar exercises but with the characteristic of being EMP, not even tried to do them. This case gives indications that the student copied in exercises 1 and 2.

Justifications for possible criticisms.

1_ How do you know they were copied? Maybe they didn't have time to finish.

The students had 1 hour to solve 4 exercises, where in reality they were two repeated exercises because two were EMT and the other two were EMP.

2_ They were not copied, they simply did not know how to do exercises 3 and / or 4.

Exercises 3 and 4 asked to find the same variables and provided the same variables as exercises 1 and 2; how is it possible that with the above, they went well in the first two and the last two even tried to do them knowing that it was a task that would qualify.

3_ Perhaps they have difficulty in identifying and replacing the list number in the EMP, so they did not do well in relation to the first two exercises.

The EMP exercises have been working since last semester in the subject of basic algebra and from the first part of the subject of geometry and trigonometry; At this point the handling of the variable "L" is not a problem.

4_ They did not want to do them, they were conformists.

At the beginning they were explained that each exercise had a value of 2.5 points making a total of 10 points for this task; It is very unlikely that any student did not want to do the EMP exercises on a whim, putting their grade at risk.

INTERVIEW WITH ALL STUDENTS WITH THE "COPIÓ" OPINION

In order to obtain a more precise result, each student who obtained an “Copied” opinion was interviewed so that from the interaction in relation to their participation in the exercises, possible differences and similarities with the results of the table can be detected.

EMP MODEL CONTRIBUTIONS

• Decrease the practice of copying.

• The student is more committed to their individual learning.

WEAK POINTS OF THE EMP MODEL

• The teacher takes longer to rate exercises under the EMP model.

• Students have marked differences in the digits they use, for example, generally the student with list number 1 has small and easy to manipulate values in relation to a student with list number 37.

5. Conclusion

At the beginning of the investigation, the objective was to analyze the contributions of this model; The research shows that one of these contributions is that the EMP model is a teaching strategy that reduces the copy in a math class with similar characteristics (number of students, educational level, willingness of the teacher, etc ...) to the class where the instruments were implemented.

The percentage of students who copied (that is, who exactly reproduced the procedure and / or response of an exercise) in traditional mathematical exercises (EMT) is 51.7% taking into account that of the total of 29 students there are 15 students who copied.

The percentage of copy decrease is expected to always be 100% taking into account that a conscious student would not have to copy an EMP from another classmate since the data and responses are different; however, there are always clueless students as in the case that occurred in the implementation of the 4 exercise sheet. In the experiment there were two students who copied (reproduced exactly the same as another classmate) in the EMT and in the EMP, therefore, the percentage of decrease of the copy is 86.7% taking into account that of the 15 students who In the traditional mathematical exercises (EMT), there were two students who copied in the personalized mathematical exercises (EMP) while the remaining 13 did not.

The practice of copying (reproducing exactly something) tends to decrease by approximately 100% since a student would not have to reproduce the procedure and / or response of another classmate because the data is different; however, there are rare cases in which the student will continue copying as in the experiment performed where two students reproduced the procedures, results and errors of another classmate even though the exercises were personalized mathematical exercises.

It was observed that there was a decrease of the "copy" by 86.7% as detailed at the beginning of this section.

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Modular arithmetic in high school math classroom

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Abstract

In this paper we propose the incorporation of congruence or modular arithmetic in the first semester of high school to motivate and involve students in mathematical activities. The aim of this work is the active participation of students and the use of numbers. Students will be able to recognize the historical importance of congruences and how it helps in numerical calculations. Modular arithmetic can also be considered as a starting point of view in the use of algebraic equations. On the other hand, the development of the numerical sense can be promoted by the use of different forms and functions of the number.

Key Words: Congruence, Period, Cycle, Calendar, Number

I. Introduction

Gauss proposed a new formalism based on his method of congruences and created the modular arithmetic in 1798. In the book *Disquisitiones Arithmeticae* (in English the title is *Arithmetical enquiries*) Gauss presents new results in number theory and expands the results previously obtained by great mathematicians such as Fermat, Euler, Lagrange, Legendre and others. From the results presented by Gauss it can be established that congruence or modular arithmetic is the most appropriate classification of rational integers 0, ± 1, ±, 2, ± 3 ... in a finite number of classes in number theory. It can be established for

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practical purposes that congruence can be considered as a statement about divisibility put into a more formal framework.

II. Some basic concepts

The basic definition that Gauss offered in 1801 about congruences in terms of divisibility can be established with a positive integer m. If a, b are integers such that m divides a-b then **a** is congruent to **b** modulo **m** and denote this by $a \equiv b \ mod \ m$. If m does not divide a-b then **a** and **b** are incongruent modulo **m**. If $a \equiv b \ mod \ m$ then b is called a residue of **a** modulo **m**. Given a ∈ ℤ , the set of integers {b ∈ ℤ ; $a \equiv b \ mod \ m$} is called the residue class for a modulo m. Congruence is a typical example, and historically the first, of the modern methodology of ordering an infinite totality into a large finite group (Bell, 2012). For example, the introduction of the concept of congruence and the appropriate notation made the treatment of linear, quadratic and higher Diophantine problems formally and theoretically coherent (Bullynck, 2009). Describing and demonstrating many of the topics in number theory would be difficult or impossible without using that concept (Kamarzarrin, Ehsan Hosseini, Hadi Mehdi Zavareh, & Kamarzarrin, 2015).

III. Modular arithmetic for whole numbers

According to Ferreira and Caballero (2016) in mathematics, modular arithmetic is an arithmetic system for integers, where numbers "go around" after reaching a certain value called module. So the idea of modular arithmetic is simple, instead of having infinite numbers, as in the case of natural ones, we are left with only a finite number of them (Vinuesa, 2011). From the point of view of Paar and Peizi (2010), most of the calculations in which modular arithmetic is involved (as is the case with cryptosystems) are based on sets of numbers that are: a) discrete (sets with integers are particularly useful y b) finite (that is, if we only calculate with some of the many numbers). A clear example can be found in modular arithmetic with integers, sometimes referred to as “clock arithmetic”, where numbers “wrap around” after they reach a certain value (the modulus). Figure 1 shows that no matter how much we increase numerals at the hours that appear on the clock, the set established on the clock face is never abandoned. The number 12 behaves like zero, because adding 12 hours to any time (again, ignoring A.M. or P.M. differences) doesn’t change anything.

Figure 1. Set of finite and discrete numbers present in a conventional clock

IV. Modular arithmetic as a calculation tool in the classroom

Despite their undeniable importance modular arithmetic is largely absent from school curricula. In one of the few investigations in the teaching of modular arithmetic Gómez (2011), has reported that the work done in the math classroom with respect to modular arithmetic has not been adequate. [He concluded that] the modular arithmetic has been taught to students mechanically and without relation to their real life. Many students find modular arithmetic is irrelevant to their pursuits and without any relation to other areas of knowledge. However, for us modular arithmetic is useful for teaching and learning mathematics because it provides a powerful introduction to the number sense: It serves to approximate large numbers, estimating answers, as a tool for developing patterns, as a resource to recognize structures, as a context for developing proofs and as a gateway to algebra. If the student is to approach numerical tasks, and apply algorithms with understanding, it is necessary that he or she become knowledgeable about the structure of number and recognize its form and functionality.

In this investigation taken to the classroom, students were introduced to the use of the number in modular arithmetic in their form of *class representative*, that is, in their form of *sets* and *subsets*. The concept of set can help students understand some topics in modular arithmetic. Students learn that a set is just a collection of numbers. Sometime the members of the set can be related to each other in some way (as is the case in modular arithmetic), although they don´t have to be. Modular arithmetic is very useful for describing cycles related to the time and frequency of an event such as calendars.

The convenience of using modular arithmetic over usual arithmetic is exposed by Yamaguchi (2009) when it shows the distinction between division by 7 in usual arithmetic, which is relevant to calendar calculation, and division in modular arithmetic with *mod 7*. The calendar is naturally expressed in modular arithmetic with mod 7, in which division by 7 corresponds to division by 0 in usual arithmetic and is not allowed. Additionally, Ethridge and King (2005) reported that calendar math involving children in experiences with concepts such as patterning, sorting, and seriating can be beneficial in a meaningful, socially constructed learning context.

V. Calendar

Around two thousand years ago in Egypt, Babylon, China, and Greece, it was known that the year takes 365 days and a fraction. According to Winlock (1940) the credit for the discovery that a solar year consisted of $365^1/_4$ days was given by classical authors to Eudoxos of Knidus (408-355 B.C.). The calendar implemented in the reign of Julius Caesar--named "Julian" in his honor--introduced a leap year every four years, putting 365.25 days in the year (Peters, 2013). So there are 1461 days in four years instead of 1460, which is the reason why we add a day every four years. In the Gregorian calendar, a normal year consists of 365 days; a "leap year" of 366 days is used once every four years to eliminate the error caused by three normal years. Any year that is evenly divisible by 4 is a leap year.

The Gregorian calendar stipulates that a year that is evenly divisible by 100 is a leap year only if it is also evenly divisible by 400. So 1900 is not a leap year, while 2000 is. The Gregorian calendar has a cycle of 400 years, with average length of the year equal to 365.2425. This is about 27 seconds longer than the current value of the tropical year. There are various estimates for when this will accumulate to an error of one day. Some people, however, have suggested that we should adjust the rules by saying that years divisible by 4,000 are not leap years (Aslaksen, 1999). Also it is well known that for each natural year (not leap) the days of the week are traveled one by one, while for a leap year (366 days) the days are covered in two units (Velasco and Antoniano, 2012). If arithmetic is used to explain the shift in two units of the days of a leap year, 366 is congruent to 2 in module 7 ($366 \equiv 2 \; mod \; 7$). Therefore, if a leap year is considered first, we can establish the following sequence for subsequent years: $2, 1, 1, 1, 2, 1, 1, 1, 2, 1, 1, 1, 2, 1, 1, 1$ ... In which a leap year cycle is repeated (represented by the number 2) followed by three natural ones (represented by three ones).

VI. Uses of the number: A Socio-epistemological perspective

The socioepistemological theory explains the problem of construction of mathematical knowledge. The phrase "social practice" refers to the activity of the human being on the environment in which it operates. According to Camacho (2006) there are two ways in which practical activity is manifested: one is the action of nature and the other, the social practices that human beings exert on knowledge. From the perspective of socio-epistemology, in the first instance it has been tried to place the subjects in a learning situation, creating tools through arguments contained in the basic algebra and the study of curves that involve processes that allow the acquisition of graphic languages that facilitate the transfer between various conceptual fields, virtually alien to traditional education.

However, there is a lack of research about the number, its forms and its functionality. Fregueiro (2014) presents one of the first investigations on the use of the whole number from a socio-epistemological perspective, concluding that the uses of the number could be evidenced from its forms and functionalities, which are presented in accordance with a certain human group and certain situation. Dominguez (2003) pointed out that the uses of knowledge are debated between its functionality and its form, therefore knowledge acquires specific form and functioning in each situation. According to Filloy, Puig and Rojano (2008) the use of number appears in very diverse contexts. The uses of numbers in each of these contexts follow rules. The totality of these uses of numbers in all contexts constitutes *the semantic field of number*, the *encyclopedic* meaning of number.

The identification of the context in which number is being used enables someone who is reading a text or receiving a message to abide by the semantic restriction that the context establishes and thus interpret it appropriately. However, the person who reads a text or has to interpret a message does not operate in the whole encyclopedia, i.e. the use of number is highly personalized and is related to what ideas about number have been established and also on how those ideas were established. It is essential that as teachers we make the choice of appropriate contexts so that a construction of relevant meanings is really favored. From this perspective, schooling plays a crucial role not so much because it teaches children new arithmetic techniques, but also because it helps then draw link between the mechanics of calculation and its meaning (Dehaene, 1997).

VII. The number Sense

Number sense refers to a person's general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgements and to develop useful strategies for handling numbers and operations. It results in an expectation that numbers are useful and that mathematics has certain regularity. The acquisition of number sense is a gradual, evolutionary process, beginning long before formal schooling begins. There is evidence that the context in which mathematical problems are encountered influences thinking (McIntosh, Reys, and Reys, 1992). It is necessary to provide students with rich situated activities which not only promote problem solving but stimulate different components of number sense.

When students understand the meaning of numbers and their relationships and structural properties, they may also be able to comprehensively develop the meaning of arithmetic, algebraic and geometric operations. Therefore, the practice of numbering in the classroom should focus on strategies that develop flexibility and fluency of numbers. The National Council of Teachers of Mathematics Curriculum and evaluation standards state that children with good number sense: have well understood number meanings, have multiple interpretations and representations of numbers, recognize the relative and absolute magnitude of numbers, appreciate the effect of operating on numbers, and have developed a system of benchmarks to consider numbers (National Council of Teachers of Mathematics, 2000). For us the number sense has to go through phases, which are: synthesis and characterization, calculation, analysis and judgment of the number, through argumentation and modeling.

VIII. Method

We propose that a study of the modular arithmetic of calendar dates, would give students an opportunity to understand the number use in their form of *class representative*. The participants were first year high school students. Modular arithmetic was presented through the development of concepts and applications to integrate and reaffirm arithmetic operations module 7. Sepulveda and Tinoco (2000) have concluded that modular arithmetic should be present from the level of basic education to higher education levels. The *development of the numerical sense* and *the use of the number* were also build on the development of social skills, problem solving, concepts, in order to provide a stimulating framework for the associative-learning tasks. One of these tasks is presented below:

Intentional activity

Johann Carl Friedrich Gauss was a German mathematician, astronomer and physicist who contributed significantly in many fields, including number theory, mathematical analysis, differential geometry, statistics, algebra, geodesy, magnetism and optics. Gauss was born on April 30, 1777 and died on February 23, 1855. On what day of the week did his birth and death occur?

We could solve the i*ntentional activity* by taking out a cell phone with a calendar and looking for Gauss's birth and death years, but a simpler method is to use the fact that the days of the week recur in cycles of length 7. Now $365 = 52 * 7 + 1$, so the first of January and the last day of December will fall on the same day, and this is easy to determine. Here we chose the modulus m=7, and replaced 365 with its remainder on division by 7, namely 1. In another example, suppose we want to determine on February 15 which corresponds to the number 41. We have $46 = 6 * 7 + 4$. In this equation, 46 is the dividend, 7 is the divisor (modulus), 6 is the quotient, and 4 called the remainder or residue.

Stage 1

This type of activities allows students to solve it from different perspectives. One of the most recurring includes the use of the number in its form of *visual representation*, where the number is used as a resource, to characterize, synthesize, calculate, reason, arrive at the solution and give a judgment. The number as a *visual representation* is used to find the first clues to reach the solution of the problem and then operate symbolically. The days of the week corresponding to January 1 are correlated with those dates in the calendar that respond to the seven-day periodic equivalence class and whose equation is: $1 + 7k$. In this equation, k=0, 1, 2, 3 …52 (for a year of 365 days). Figure 2 shows the equivalence classes for the year 2019, where the first of January was Tuesday. From the 2019 data, students needed to map the days of the week to numbers in a module system. There are 7 days in a week, so the students used the module 7 to build the mapping.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
0	1	2	3	4	5	6

The equivalence classes that were associated to January 1 are: {01, 08, 15, 22, 29, 36, 43, 50, 57, 64, 71, …, 134, …, 204, …, 344, 351, 358 y 365} that correspond to calendar dates: {January 1, January 8, January 15, January 22, January 29, February 5, February 12, February 19, February 26, March 5, March 12, …, May 7, …, July 16,…, December 10, December 17, December 24 and December 31. The students perform the modeling and establish the structural pattern of the calendar that implied the recognition of the association of the type numbers: $01 + 7k$. Additionally, students identified in Figure 2, that day 54 corresponds to February 23 which is the anniversary of Gauss's birth (Saturday) and that day 120 corresponds to April 30, date of Gauss's death anniversary (Tuesday). From this analysis, students establish correspondence between the first day of the year and any date by assigning it a day of the week. In these types of tasks, the objective was to recognize the visual pattern, try to describe it and find the rule that allows explaining what was identified, the way they performed it and the conclusions they reached.

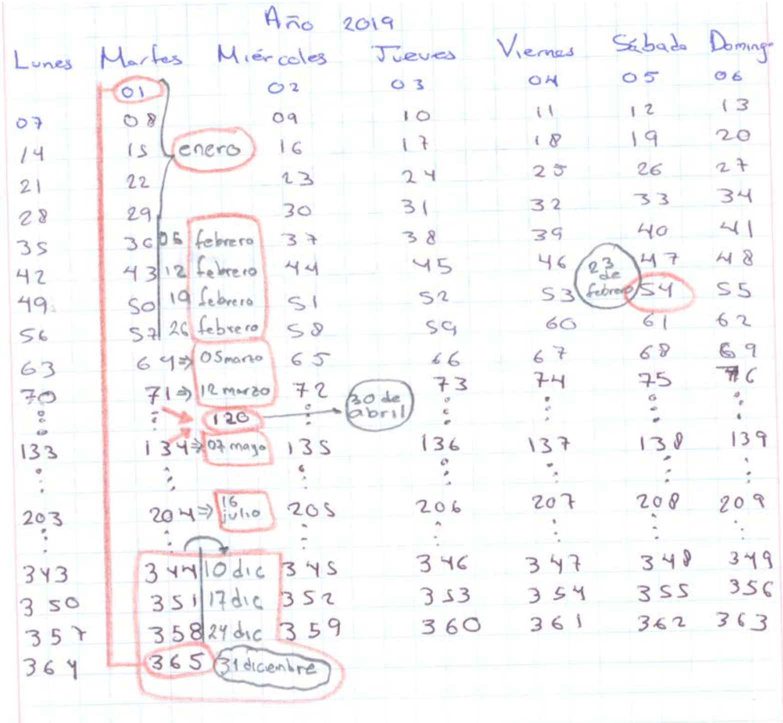

Figure 2. Use of the number as a visual representation

Stage 2

With a little math, teacher help and modular arithmetic, students were able to determine the exact day of the week of any date: past, present or future. In the math classroom it was established that, like the numbers on a clock or the number of days in a week, counting numbers in modular arithmetic repeats an established pattern. Using a *visual representation* of the number, the determination of the day of the week was a simple and quick calculation; with a little practice, students were able to do it accurately without paper, pencil or calculator. Figure 3 shows the tables of values assigned to the years 2019 until Gauss's birth date, which were used for reflection and discussion about *what is maintained*, and *what varies* in the determination of the day assigned to the first of January in the period of time studied, which promotes ***variacional thinking*** in students.

The analysis performed by the students began in the most recent leap year (2016) until the leap year closest to the birth of Gauss (1780).

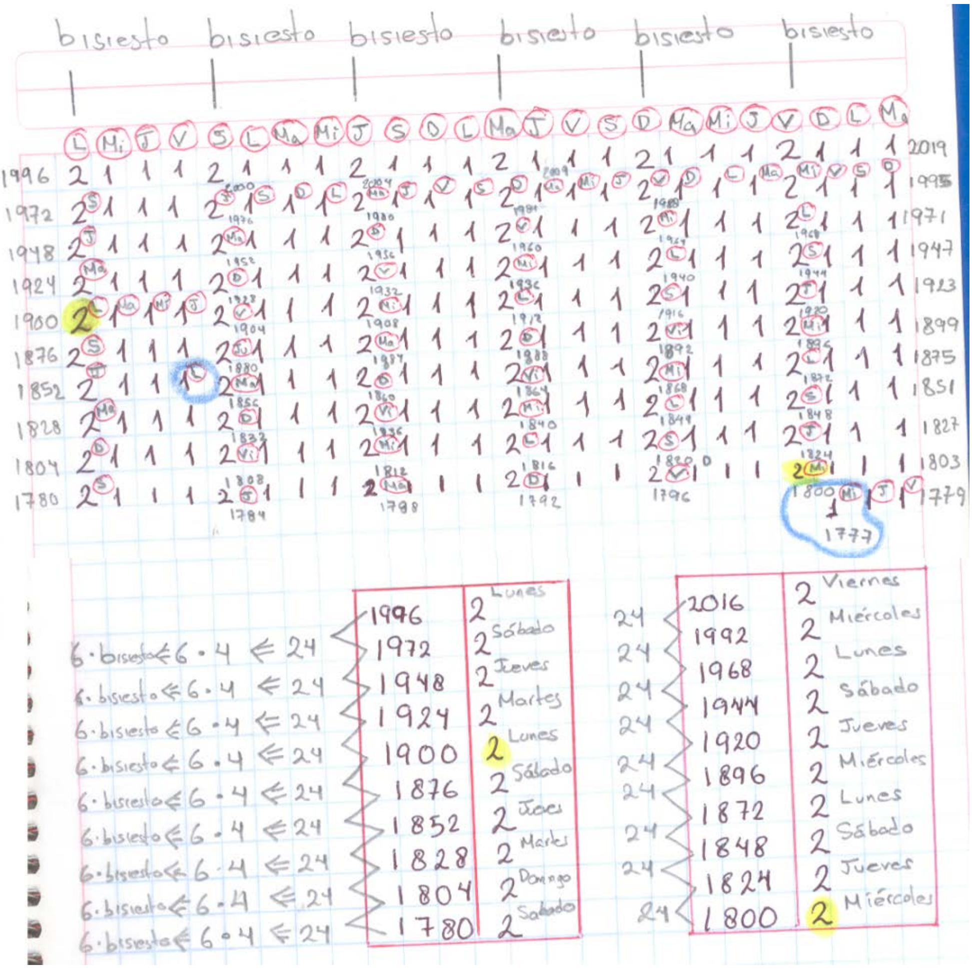

Figure 3. Use of the number and its representation as a value table

For the classification of leap years, according to the arrangement described in Figure 3, they were accommodated in such a way that in each of the rows there are 6 leap years and as each of them has four years, in each row there is a difference of 24 years. In Figure 3, you can see the shift of the days of the week in a period of 24 years, that is to say the shift of the days of the week that can be verified in each of the rows, it is possible to highlight two years in particular 1900 and 1800 that at first glance one might think that they are leap years and that in reality they are secular years not leap years because they are not divisible by 400.

Stage 3

The students performed the analysis for the years 1777 and 1855 to give a numerical answer using the number in their ***visual representation*** for the day of the week of the birth and death of Gauss. In Figure 4, the use of the number as a visual representation is shown. For students, this use of the number had the functionality of organizing their ideas, recording data in an orderly manner, and giving the answer to the proposed intentional activity.

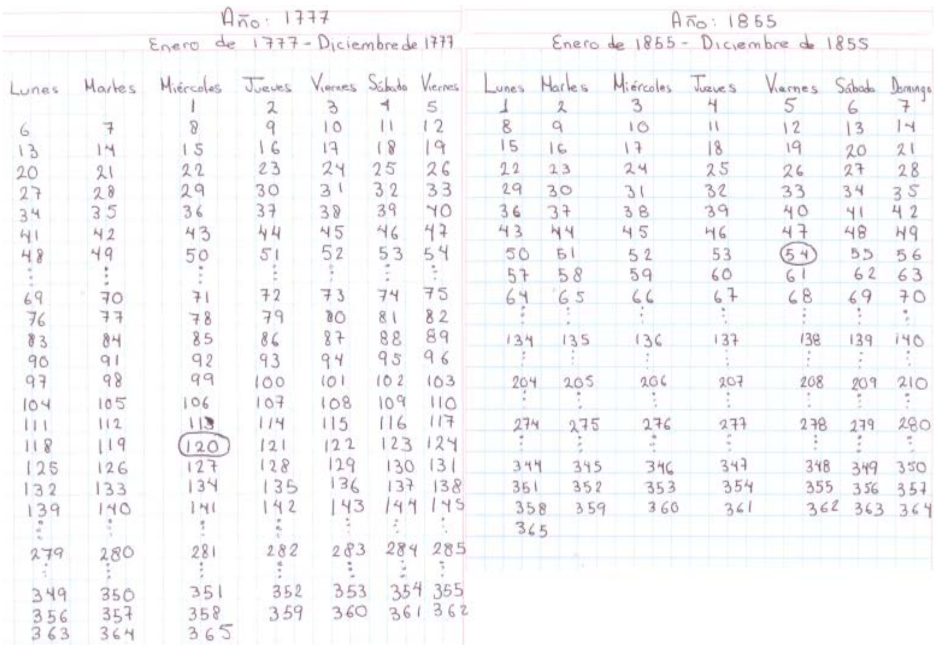

Figure 4. Use of the number to determine the day of Gauss's birth and death

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Stage 4

In the previous three stages it was possible to identify the appearance of regularities in which structural patterns are revealed in the determination of the day of the week in a particular year. In these regularities, the *processes of generalization and particularization* that are fundamental for the presentation of modular arithmetic in the classroom were worked on. Students apply the relation of congruence that Gauss introduced in section 1 of the book Disquisitiones Arithmeticae,

$$a \equiv b(mod\ m) \ \Leftrightarrow m\ divides\ (a - b)$$

The Latin word "module" means "measure" or "scale", for the calendar case the "measure" or "scale" is 7. In Figure 5, it is shown that $54 \equiv 5\ (mod\ 7)$, and $120 \equiv 1(mod\ 7)$, so that in any given year, which is being analyzed on February 23 will fall on a day of the week identical to January 5 of that year, while for April 30 of a non-leap year, it can be established that the day of the week will be the one that corresponds to January 1 of that year.

Towards an extension of calendar arithmetic

The arithmetic of the calendar can be very useful for students in their search for numerical relationships and the analysis of change situations, and can be perfectly complemented by visual representations of the number, graphics or images. The idea behind congruences had been known and exploited for a long time: residue classes mod m were familiar as ‘arithmetic progressions with common difference m’, and congruences were used as a tool in the solution of equations in integers. In this part of the proposal the student is able to understand these phenomena, describe them, analyze them and model them. It is about taking advantage of what was learned in the activity “birth and death of Gauss”, and generalize it for any date of any year, for this purpose it was chosen to do it through a playful activity in the form of a game, with the intention of enhancing skills that allow them to identify regularities, understand the concept of congruence and analyze the concept of ***invariance***.

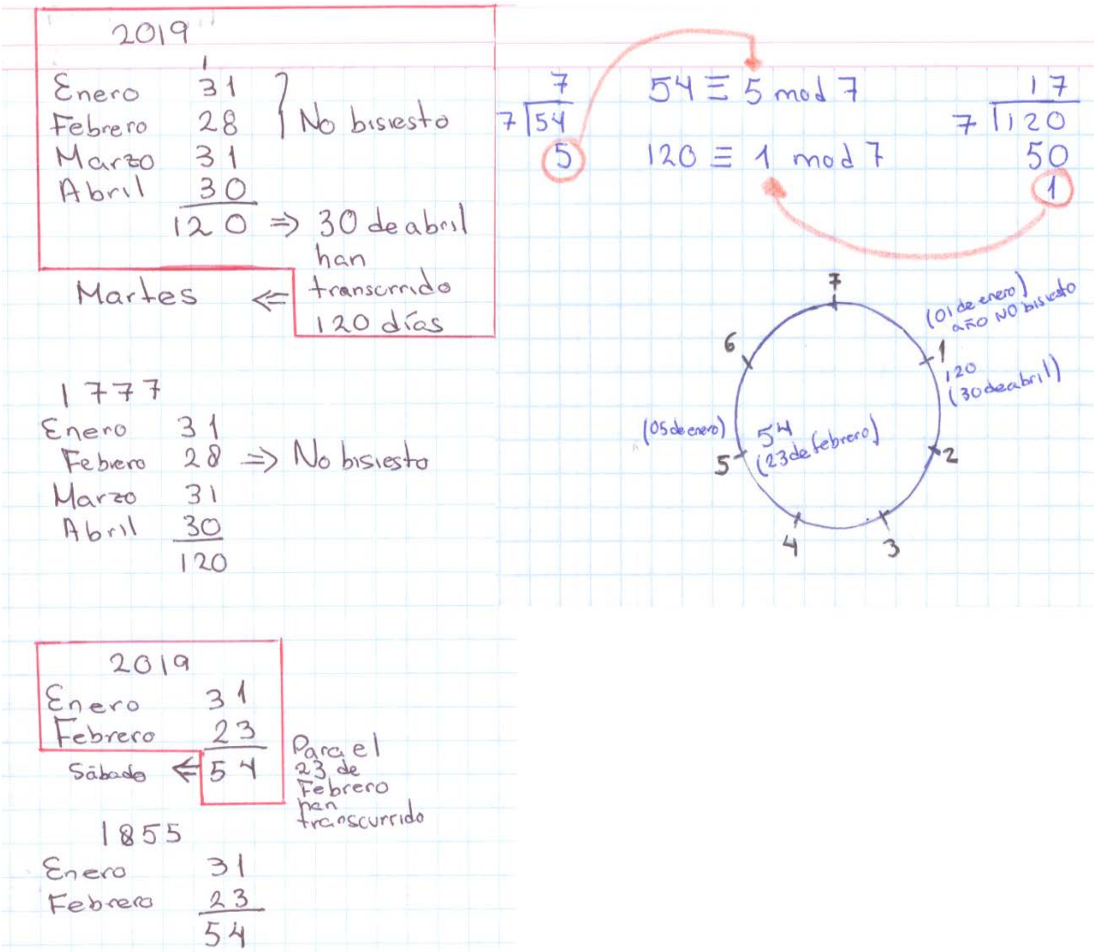

Figure 5. Use of modular arithmetic and use of the relation of congruence

The game that was presented to the students was called "Project Almanac", which refers to a fiction film about time travel. Time travel became part of the game having an important and fundamental role in obtaining the dates that would be used in the game; these dates were chosen from a large number of films from which 24 were selected. The general steps of the game are described below.

1. In order to provide some familiarity of the students towards the “Almanac Project” game, they are invited to participate considering their similarity with board games that are well known in Mexico, where it is possible to advance through the 24 boxes.
2. The first box is reserved for the movie “Project Almanac” (from which the name of the game was taken) and was the starting point of the game.
3. Each of the boxes corresponds to a particular movie and there are assigned cards in the form of questions that students will have to answer using modular arithmetic, such as; establish the day of the week in which a transcendent event occurred either for the characters or for the same movie with data such as the year and month of such event.
4. The boxes corresponding to the prime numbers are forward or backward and have specific instructions where students will have to break down numbers into their prime factors and determine the number of them.
5. The first of the students to arrive exactly at box 24 will be the winner and the second, third, fourth, etc. places are determined according to the arrival at box 24.

It is recommended that the game “Project Almanac” be carried out with a small number of students for control reasons.

Results

With this type of intentional activity, students are encouraged to explore the modular arithmetic that although they are present in their daily lives is not something they are aware of and even less think that it may be present in something as ordinary as a calendar is. We completely agree with the conclusion of Blasco (2016) that knowing the day of the week of a given date interests everyone, for example on the day of birth or birthday. The determination of the day of the week of the dates on the life and death of Gauss, presents an example of a sequential pattern, in which a visual representation of the number was used as a tool for its analysis. This process generates a variety of descriptions that demonstrate students' abilities to identify and explain numerical relationships. But it is also useful; to determine the position on the calendar of designated dates. Through the game "Project Almanac" students faced the calculation of the day that the card asked them according to the film that corresponded to each box, so they also expressed interest in seeing what the film was about or in some cases recognizing her. The students worked collaboratively supporting each other and discover that the exchange of ideas and knowledge is very necessary to be able to advance in the corresponding boxes (Figure 6).

Figure 6. Students applying modular arithmetic in the game: "Almanac Project"

It is possible, therefore, as described by Blasco, Duran, Torrent, Castaño and Marc (2017) to establish the utility to deduce, with very simple mathematics, how the lag of one day of the week occurs in each year and how every four years we must introduce another day of lag. In the game "Project Almanac" different representations of the number appear in contexts where modular arithmetic is present in the search for the development of the *numerical sense*. Thus, we put our interest in not introducing students to modular arithmetic through meaningless definitions, or the application of rules, formulas and algorithms, but in the use of the number and its representations by students to promote the development of the numerical sense. In addition, that the game becomes an excellent tool to bring students closer to modular arithmetic, it also offers us an opportunity to strengthen the bonds of fellowship in the classroom in order to learn by living together.

Conclusions

Using the historical context coupled with a playful presentation of mathematics can contribute to students approaching the modular or arithmetic of congruences and develop the numerical sense giving it a deeper meaning in search of an integral mathematical thought, in contrast to the application of meaningless rules, formulas and algorithms. We agree with Blasco et al. (2017) when they talk about the relevance of introducing activities in which modular arithmetic appears, which they consider key today in computer security, as well as in other fields of interest. In activities such as the one presented in this work regarding the birth and death of Gauss and in general the arithmetic of the calendar, the objective is to reason and draw conclusions, first personally and then discussing in groups. So the arithmetic of the calendar and its school context encourages students to use the number with the intention of acquiring skills in the area of mathematics, also reasoned, conjectured, discussed and defended their ideas. Students integrated knowledge they previously possessed with those they acquired through the use and management of modular arithmetic, favoring cooperative work and encouraging the development of problem solving strategies.

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Opportunities and challenges for project-based learning: the case of statistical projects

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Abstract

Based on a review of literature related to work with statistical projects, this article discusses aspects of the use of statistical projects for the teaching and learning of statistics, as well as the challenges and opportunities involved in implementing them. The contributions of this article include an overview of work with projects which examines the five criteria that all projects should consider in order to promote project-based learning. In addition, recommendations for instructional design are given, together with reflections that seek to encourage teachers, both present and future, to use project-based learning as a modern, efficient option for teaching and learning statistics.

Keywords: Statistical projects, project-based learning, teaching and learning statistics.

Introduction

According to Knoll (2012) and Pecore (2015), the origins of project-based study can be traced to the early 18th century in Paris where, Knoll mentions, advanced students at the Royal Academy of Architecture (*Académie Royale d'Architecture*) had to demonstrate their knowledge and abilities by planning construction projects (e.g. churches, monuments, palaces, castles). In 1865, the Massachusetts Institute of Technology (MIT) became the first educational institution to utilize projects as a teaching method. Ulrich (2016) tells us that

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Professor John Dewey was the educator who, in 1910, proposed projects as a way to "learn by doing", based on students' research interests and a constructivist focus. Knoll (2012) reports that in 1914 the U.S. government incorporated project-based learning (PBL) in vocational schools, though it was not until 1918 that this approach truly came into vogue with the publication of the document *The Project Method* by William Heard Kilpatrick. This author considered that motivated students to develop their abilities and obtain high levels of knowledge, while leading them to appreciate, enjoy and trust school activities (Pecore, 2015). Kilpatrick (1918) wrote that projects are intentional acts and that, in his view, project-based education better prepares students for life because intentional acts constitute, in and of themselves, life experiences. While the idea of using projects as a teaching tool predates Kilpatrick, it was his dissemination of a technique for applying them in educational contexts through his group ("the project propaganda club", Knoll, 2012) that fostered the recognition that projects now enjoy in the field of education. In the early 1990s, the project method as a teaching/learning model was formalized in a focus called Project-based Learning, understood as "a model that organizes learning around projects" (Thomas, 2000, p. 1).

In the field of statistics, projects are recognized as being especially useful in changing the traditional way in which this discipline was studied (Sagarribai, 2015). According to research on statistics education, the use of projects is judged an effective teaching method for professors and an effective learning tool for students (e.g. Baglin, Bedford & Bulmer, 2013; Bailey, Spence & Sinn, 2013). Today, however, statistical projects still tend to be employed more as a means of evaluating students' learning (American Statistical Association [ASA], 2016; Garfield & Ben-Zvi, 2008) during, or at the conclusion of, courses (though the latter case is less frequent, Biajone, 2006). In addition, projects are often used as a resource for reviewing topics already studied in class or during laboratory practice (Biehler, 2007).

In light of the potential of the PBL approach, the objective of this article is twofold: first, to describe approaches to the use of projects as a method for studying statistics and, second, to identify the opportunities and challenges entailed in using this method. To achieve this, it presents two sections related to: (1) the use of projects in the process of teaching-learning statistics; and (2) the opportunities and challenges associated with this tool. The first section describes issues that arise when implementing projects in the classroom, the phases involved in organizing work with projects, and the activities included in each phase. The second discusses the opportunities and challenges involved in implementing this model. Finally, a section on final considerations offers a vision of the criteria that should be taken into account when employing work with projects as a true method for encouraging learning.

Method

This study utilized the documental research method, which consists in analyzing documents that contain information on the study topic (Bailey, 1994). The publications reviewed relate to work with statistical projects. The inclusion criterion applied was that, insofar as possible, they discussed details of how to implement projects in the classroom as a tool for teaching statistics. Searches were conducted in several sources, including international research journals, acts of congresses, and Master's and Doctoral theses. The review centered on publications between 1994 and 2017 and included documents in Spanish and English. The search for information was conducted on the Internet and produced a total of 77 texts dealing with statistical projects.

The use of projects in the process of teaching-learning statistics

According to Batanero and Díaz (2005), statistical projects (SP) constitute a specific type of research in the field of statistics that scholars seek to integrate into broader research processes. Young (2017) states that SP entail a broad process of activities that begins with a problem in some branch of knowledge, or a research question that will be addressed using statistical methods and ends with a written report that presents the findings observed. Hence, SP consist of a series of systematized activities designed to respond to a research question using statistical techniques, and then present results in a written report or poster. It is important to insist that the results of projects need to be disseminated to as many people as possible, either orally or in the form of a poster, in order to share information and demonstrate the potential of applying statistics to problem-solving (Bailey, Spence & Sinn, 2013).

Statistics education recognizes the significance of SP from various perspectives: as an important tool for evaluating aspects of students' learning and helping them experience different stages in proposing and solving a statistical problem (delMas, Garfield, Ooms & Chance, 2007; Franklin et al., 2007; Garfield & Ben-Zvi, 2008; ASA, 2016); as a strategy for teaching statistics (Batanero and Díaz, 2005; Ojeda, 2011); as an ideal medium for providing learning experiences and reflecting on research in statistics (MacGillivray & Pereira-Mendoza, 2011; Makar & Fielding-Wells, 2011); and as a method for fomenting the development of a culture of statistics (Conti & Carvalho, 2014), statistical reasoning (Sovak, 2010), and statistical thought (Binnie, 2002).

Before involving students fully in SP activities in the classroom, it is suggested that teachers devote at least five hours of their courses to constructing an effective framework that provides guidelines for students to clarify the activities to be performed and how to perform them. The approach may include showing examples of SP, explaining what students are expected to do, presenting the study program, outlining the learning goals, organizing the work (individually or in teams) specifying the system of evaluation, and elucidating the action plan. Once this framework is established, students can begin to develop their SP.

Findings on the implementation of SP

It turns out that SP are usually implemented towards the end of courses and that they have an average duration of around two months, though they may last a full semester or begin with the introductory phase of the course (Verhoeven, 2011). Usually, only one project is developed during a course (Baglin, Bedford & Bulmer, 2013), but in some cases as many as three have been planned (Sisto, 2007). While SP can be developed on individually (Carnell, 2008), they are more commonly conducted in teams of 3-to-5 members (Nascimento & Martins, 2008).

In terms of evaluation, SP are typically monitored continuously throughout their development, and their products are graded using typical evaluation tools (Figueroa, Baccelli & Prieto, 2014). The evaluation process is normally performed by the teacher (Ghinis, Chadjipantelis & Bersimis, 2005), but it is also possible to involve students in assessing the work of their classmates (Bulmer, 2010). In some cases, all projects carried out are presented orally to a group (Santos and César, 2006) using a scientific cartel or poster (Halvorsen, 2010). A scientific poster is a printed presentation in large format (e.g. 90 x 120 cm) that presents a visual summary of the fundamental aspects of a project (*i.e.*, a problem or methodology, results or conclusions). Cartels can be read by the attendees and/or presented verbally by the author. After performing partial or final evaluations of students' work, it is necessary to give them feedback that encourages them to reflect on the strengths and weaknesses of their projects, and then make the required corrections to improve them.

Phases of the implementation of SP

Diverse sources include discussions of the process of developing statistical projects (Wild & Pfannkuch, 1999; Batanero & Díaz, 2005; ASA, 2016), but there is no broad agreement on the criteria for determining the number of steps involved. In general, however, seven phases can be identified (see Figure 1), and this scheme can serve as a reference for understanding how projects may be implemented.

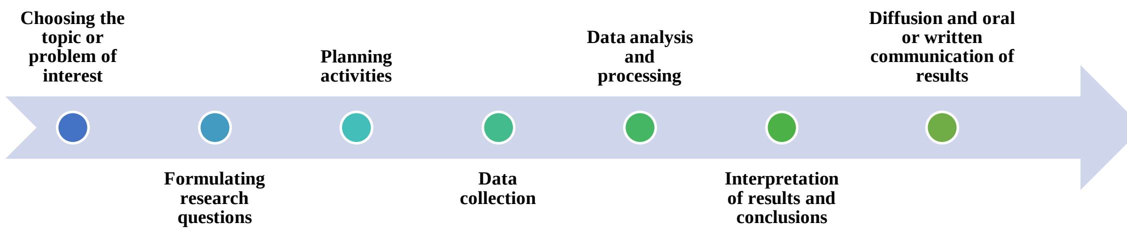

Figure 1. Phases of a statistical project

The following section presents a general description of representative activities for each one of these phase of an SP (Table 1).

Table 1.
General description of activities in each phase.

Phases of the project	*Description of activities*
1) Choosing the topic or problem of interest	✓ Adequately define and delimit the study topic, or problem chosen, for the project.
2) Formulating research questions	✓ Formulate the statistical questions that will orient the project

Santana, A., & Gómez-Blancarte, A. (2019). Opportunities and challenges for project-based learning: the case of statistical projects. In A. Rosas (Ed.), *Research Reports in Mathematics Education: the classroom* (pp. 63-82). Miami, FL: L.D. Books.

3) Planning activities	✓ Design an action plan for developing the project. ✓ Identify the data sources (measurements, simulations, databases). ✓ Define the variables, study population and sample for the project. ✓ Define the type of statistical study (*i.e.*, descriptive, correlational, inferential, comparative). ✓ Choose the statistical tests to be applied, depending on the nature of the data and the context. ✓ Determine the software to be utilized for data analysis. ✓ Elaborate the instrument(s) for data collection.
4) Data collection	✓ Gather or generate the data. ✓ Clean the data.
5) Data analysis and processing	✓ Perform a transnumeration of the data; that is, process the data in the form of tables, graphs and descriptive statistics. ✓ Carry out the analysis of the information. ✓ Generate the hypothesis.
6) Interpretation of results and conclusions	✓ Interpret the tables, graphs and descriptive statistics. ✓ Interpret the results of the statistical tests applied. ✓ Perform a holistic interpretation of the results according to the context of the topic. ✓ Respond to the statistical questions and propose solutions to the statistical problem examined. ✓ Posit new research questions.
7) Diffusion and oral or written communication of results	✓ Write the final report. ✓ Elaborate a scientific poster to summarize the results of the project. ✓ Present findings to the student community and, if possible, to broader audiences.

While the number and type of activities conducted depends on the nature of the project (e.g. applying a survey or performing an experiment), the descriptions offered above can serve as a guide for orienting work with SP.

Opportunities and challenges of implementing statistical projects

Our review discovered a series of opportunities and challenges that underlie the design and implementation of SP. These are analyzed in the following section.

Opportunities

Working with SP offers opportunities to both teachers and students. For teachers, projects allow them to attract students' attention and provide a means to achieve high levels of motivation (Jolliffe, 2002). Also, they can take statistics out of the classroom to contextualize their application and demonstrate their usefulness (Verhoeven, 2013). Experiences with SP can also help consolidate the teaching abilities necessary to ensure that activities proceed positively, such as the professional commitment to continue to utilize PLB as a primary teaching strategy (Makar, 2008).

Turning to students, SP can improve the way they learn by emphasizing collaboration (Walsh, 2011) and providing opportunities to become familiar with, and utilize, the technologies required to develop PLB learning (Spence & Bailey, 2015). Students also have the opportunity to broaden their knowledge and enhance their abilities in, and attitudes towards, statistics courses (Villazcán, 2014) because working with projects transforms them into protagonists of their own learning process. Project work also tends to enhance oral and written communication skills (Halvorsen, 2010), stimulates curiosity, and leads to conceptual and procedural maturation in statistics (Dierker et al., 2016). Finally, SP give students the opportunity to construct their own knowledge (Ramirez-Faghih, 2012) and simultaneously encourage them to fulfill their responsibilities (Bailey, Spence & Sinn, 2013).

Challenges

In addition to benefits, however, it is necessary to consider the challenges entailed in working with SP. The challenges for teachers are associated with the amount of time required to implement this strategy (Rivera, 2015), and difficulties in administering classroom time due to the need to find a balance between their own participation and spaces of autonomy for their students. This is a significant challenge because teachers must focus on the issues involved in guiding the procedure while also linking the topics that students need to learn with the diverse activities included in their projects. This is because the goal of using SP is to stimulate students to construct their own knowledge through those activities, instead of simply receiving it from instructors who teach topics in the traditional way.

Moreover, using SP means designing alternative systems of evaluation (*i.e.*, beyond standardized written tests) to assess the knowledge and abilities that students obtain and develop. Without doubt, the SP approach demands a huge workload for teachers that includes planning activities and elaborating an adequate framework for evaluation. These factors may limit teachers' desire use apply this strategy in the classroom (Bulmer, 2010). In addition, employing projects obliges teachers to reconsider and redesign the assessment criteria they apply during the work their students perform to ensure that they can objectively measure the advances that students make in successive stages of their projects (Nascimento, Martins and Estrada, 2014). To effectively address some of these complexities, teachers are obliged to make ample changes in their approach to teaching (Terán and Nascimbene, 2015). Weak training in statistics or statistical didactics can impede achieving adequate results, precisely because the SP strategy demands that teachers orient, supervise and accompany the students throughout the process (Makar, 2004).

Students, meanwhile, may also confront difficulties if the teamwork involved is poorly coordinated or organized (Holmonth, 1997). Also, some may find it difficult to decide upon a study topic, especially if they are not adequately informed of their options (Wardrop, 1999), others might find that gathering and processing data are tedious, difficult tasks (Halvorsen, 2010), while still others could find the degree of complexity of the statistical topics addressed in their projects quite challenging (Carnell, 2008). During the development of SP, students will confront problems in composing significant research questions, difficulties caused by the complexities of administering their time, challenges when it comes to adequately transforming their data and processing their information, and problems in developing an operating logic to support their decision-making (Batanero and Díaz, 2005). Finally, students must understand the fact that projects oblige them to become protagonists in the activities proposed, and be keenly aware that better opportunities for learning will emerge if they embrace the responsibility and autonomy required to carry out the tasks included in their SP.

Final considerations

Although the great potential of working with SP has been documented many times (e.g. Inzunza, 2017), it is necessary to clarify that not just any project can be considered a representative example of PBL. Rather, it is fundamental to recognize and understand the characteristics that convert projects into authentic cases of PBL. Thomas (2000) sustains that true examples of PBL must satisfy the following five criteria: 1) centrality; 2) research questions; 3) constructive research; 4) autonomy; and 5) realism. The criterion of **centrality** has two fundamental elements. The first one is that the project is the principle vehicle of the teaching-learning process. In PBL, projects are the central teaching strategy because the intention is that students will encounter and learn the basic concepts of statistics through their projects. In some cases, project work is introduced after traditional instruction (formal classes led by the teacher). Here, the project provides illustrations, examples, additional practice or practical applications of the material initially taught by other approaches. However, according to Thomas' definition, *application* projects are not representative of PBL. The second aspect of the criterion of centrality rules out *enrichment* projects –*i.e.*, those with activities in extracurricular contexts normally reserved for "high achievers"– in which students learn content found outside their study plans, as examples of PBL, no matter how creative, attractive or motivating they may be.

The second criterion highlights the importance of **research questions** or problems that orient students and motivate them to discover, and struggle with, the central concepts and principles of the discipline of statistics. The questions that students pursue, as well as the activities and products to which they devote their time, must be orchestrated in such a way that they have an important intellectual purpose (e.g. cultural development, reasoning or statistical thought).

The criterion of **constructive research** signals that the central activities of projects must lead students to search for and solve problems, transform and construct knowledge, achieve new understandings, and/or acquire new abilities. Their research may lead them into processes of design, decision-making, searches for issues, problem-solving, and discovering or constructing models. But to be considered a PBL-type project, all these central activities must promote the production of new knowledge by the students themselves. If those activities present no difficulty for students, or if they can be carried out by applying information that is already known or abilities that have already been learned, then the project is an exercise and cannot be considered a true case of PBL.

The criterion of **autonomy** refers to the fact that projects oblige learners to accept responsibility and participate in decision-making. Generally-speaking, PBL-type projects are not directed by the instructors along specific guidelines, nor are they perfectly delineated so as to simplify their application and development. Laboratory experiments and procedures, or clear instructional manuals with carefully-planned sequences of activities, are not examples of PBL, even though they may focus on fundamental problems and tasks included in a study plan. PBL-type projects do not follow pre-defined routes, and they do not produce pre-determined results. In strong contrast to this, projects developed within the scope of PBL offer students much more autonomy, choice of action, unsupervised work time, and responsibility than traditional instruction or projects.

The final criterion is **realism**. This means that projects must incorporate challenging elements from real life. True PBL-type projects are not simple, decontextualized forms of schoolwork, for they must include features that imbue a sensation of authenticity in students. These may include the topic or problem itself, the tasks and activities to be performed, the roles that students play, the context within which the project is developed, the collaborators who work with students on the project, the products generated throughout the process, the public that will receive information on those products, and the criteria applied to evaluate both the activities carried out and its products (e.g. a written report or poster). PBL, therefore, integrate real-life challenges by focusing attention on solving problems or addressing authentic –not simulated– questions, where the solutions proposed must have the potential to be implemented in an effective way. Another key point is that the instructional design must consider the five criteria mentioned above which, in general terms, suggest that the project will be the central source of learning about significant topics in the discipline of statistics.

In line with these criteria, we consider that the principle challenge that teachers confront consists in successfully implementing PBL projects as the principle means of students' learning; that is, having them take on the role of the primary teaching strategy (criterion 1). Many teachers, however, struggle with this very question: how can I adopt projects as my central teaching strategy instead of using them only as a method for reaching final evaluations or for applying the topics reviewed in the study program?

The PBL approach further suggests that work with SP should begin practically at the outset of the course that the instructor is assigned, that its implementation should be prolonged as much as possible –at least one quarter, preferably a full semester– and, insofar as possible, the projects be considered the basis for evaluating the entire course.

In addition to the careful work they must devote to designing their didactic proposals, teachers must resist the urge to present topics to students before they begin to reflect on, and research them, deeply on their own. This means reducing the number of traditional classes given in front of the group. The instructors who choose to implement PBL in their courses must be ready, willing and able to delegate responsibilities and trust their students. They must make sure that learners understand, from the first, that their job will not consist primarily in "giving classes" but, rather, in orienting and supervising them as they seek to resolve the issues that will emerge during the development of their projects. In this way, students will be fully aware that their learning process will always include specific activities and numerous topics that they will have to research.

Regardless of the level at which they work, teachers will often feel that their students are not advancing, or not performing their work in the way they expect; however, they must understand that this is part and parcel of the PBL process and of the transformation that, it is expected, will become visible in their students over time.

While the PBL approach certainly offers students greater flexibility, it also transforms the role of the instructor. By implementing projects, teachers become facilitators who can overcome the limitations of the traditional class format where *they* are the only protagonists. This method equips professors to employ diverse activities that can help foment a culture of collaboration that sparks students' interest and motivates them to study statistics. Of course, teachers who accept the challenge of employing SP must be deeply-knowledgeable of the topics in the study program that they expect their students to learn during the didactic activities developed, and must have good knowledge of didactics in order to apply this methodology; if not, the experience can become extremely complicated and discouraging for both themselves and their learners.

We believe that implementing SP will –in the short term– become a common practice in classrooms, and that it is indispensable for future teachers to participate in training activities that give them opportunities to perform all the phases of an SP before they are formally integrated into their professional labors. We firmly believe that this kind of experience will help them acquire the knowledge of statistics and the teaching abilities required to achieve excellent performance when they decide to implement the PBL approach in their courses. Finally, additional positive experiences with SP will allow professors in-training to commit to implementing the PBL teaching/learning strategy in their classrooms.

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A didactic sequence for the geometric interpretation of the derivative

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Abstract

The understanding of the formula and definition of derivative of a function in a point, presents difficulties to the students of Differential Calculus in the Technological Institute of Comitán. For this reason this paper aims to propose a teaching sequence supported by the Theory of Registers of Representation of Raymond Duval on the geometric interpretation of the derivative that can help us understand both the formula and the definition of the derivative and where contributions of research in Mathematics Education; and analysis of concepts seen in textbooks are necessary for the understanding of the mathematical concept, and also with the support of a computational tool for its comprehension.

Key Words: Didactic sequence, geometric interpretation, derivative.

Introduction

The teaching and learning of calculus constitute one of the greatest challenges of current education, there is a consensus that although students can be taught to mechanically calculate derivative, primitive calculations and solve some problems, these actions are far removed of what a true understanding of the concepts involved and an adequate development of thinking methods means (Vrancken, Engler & Müller, 2010). Research on the teaching and learning of differential calculus has been carried out for several years. Artigue, Douady,

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Moreno and Gómez (1995) mention that, although students can be taught to perform some derivative calculations more or less mechanically and to solve standard problems, there are difficulties for 18-20 year olds to achieve a satisfactory understanding of the concepts and methods of thinking that make up the center of mathematical analysis. For example, some students are able to solve the exercises that are proposed to them with the correct application of the rules of derivation; however, they have difficulties when they need to manage the meaning of the notion of derivative, either through its analytical expression, as a limit of the incremental quotient, or in its geometric interpretation, as a slope of the tangent line (Ubuz, 2007; Sahin, Aydogan, Kursat, 2015; Feudel, 2019).

On the contrary, there are investigations (Hähkiöniemi, 2006; Mubaraki, 2017; Feudel, 2019) that establish that the use of different representations can facilitate the student's understanding of the concepts of derivative. Wedad Mubaraki in his master's thesis mentions that “For example, when students experience multiple representations while learning derivative concepts they made the connections between algebraic expressions and their graphs of functions” (Mubaraki, 2017, p.14), and later states that

> For instance, when students learn the derivative of functions using visual representation such as graphs… (e.g., from Rogawski’s & Adams (2015) calculus textbook), they can make visual connections between the original function and its derivative, such as the slope of the tangent line or the secant line on the curve (Mubaraki, 2017, p. 14)

In fact the geometric representation turns out to be a form still used in the calculation of functions of several variables (Trigueros, Martinez-Planell, 2010), similarly Trigueros, Martínez-Planell and McGee (2016) presented a research report on a genetic decomposition for the plane and the tangent plane in 3D, equivalent to the tangent line in 2D, using the geometric representation.

Competence number four of the Differential Calculus program within the careers of the National Technological Institute of Mexico deals with the Geometric Interpretation of the Derivative. One of the learning activities is to show students that the value of the slope of the tangent to a curve at one point can be obtained by calculating the derivative of the function that corresponds to the curve at that point. Based on the above, a didactic sequence is proposed where the student understands that the derivative of a function "It is the slope of a line tangent to a curve at a given point", through different representations for a better understanding of the mathematical concept.

The analysis of the results obtained allowed us to know the importance of establishing a didactic sequence of the content to be taught and also to define the previous knowledge that is required for the student to understand the geometric interpretation of the derivative, such as: secant line, tangent line, slope of a line, differentials, limit of a function, as well as recognizing the importance of implementing another representation of the mathematical concept through a computational tool.

Theoretical framework

Different types of educational activities have been tried for a long time to address calculation issues (Zachariades, Jones, Giannakoulias, Biza, Diacoumopoulos & Souyoul, 2007). Tobón, Pimienta and García (2010) wrote in the book Teaching Sequences: Learning and Skills Evaluation, that the current social context and the changes that are coming in the near future pose the challenge of moving from the emphasis on teaching planning, to a new teaching

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role, which entails the generation of significant situations, so that students learn what they need for their self-realization and their participation in society. So far, at the Tecnológico Nacional de México, educational programs are based on the competency model and the teaching sequences are a relevant methodology to mediate the learning processes in the framework of learning or reinforcement of competences. Which is applicable to students as well as teachers (Malaspina, Mallart & Font, 2015).

Didactic sequences allow us to carry out relevant activities, such as representing a mathematical concept in many ways, in order to improve the learning process in the student. Vrancken, Engler and Müller (2010) mention that every mathematical concept needs representations since there are no objects to display in their place and only through them is an activity possible on mathematical objects. Algebraic representations, tables, graphs and verbal expressions that contain the same information, put into play different cognitive processes, each related to the others, learning mathematics consists in the progressive development of coordination between different representations (Chang, Cromley & Tran, 2015).

For this reason, the theoretical elements to be considered are framed in the "Theory of Semiotic Registries" by Raymond Duval.

Nieto, Viramontes and López (2009) comment that to have access to mathematical knowledge it is necessary that mathematical objects be represented in different ways. Macías (2014) indicates that a characteristic of mathematical concepts is the need to use different representations to assimilate and apprehend them in all their complexity. The role that symbols play in the development of mathematical thinking is decisive, which implies, from a cognitive perspective, that for the total understanding of mathematical notions it is necessary to employ and coordinate more than one system of representation (Ruiz, 2011; Chang, Cromley & Tran, 2015).

For the geometric interpretation of mathematical concepts, graphic representation is important, so some studies recommend the use of computational tools (Diković, 2009; Ruiz, 2011), which constitute a support for the incursion of the new teaching-learning methodology, since current students are immersed in everything related to technology and can contribute to increasing motivation for the development of skills and meaningful learning (Medina, Medina y Flores, 2016).

Methodology

The facilitator is the person who plays the most important role in the task of helping students acquire skills and abilities, by designing learning scenarios. Therefore, a didactic sequence of the Geometric Interpretation of the Derivative is elaborated to strengthen the understanding of the formula and definition of the mathematical concept through the verbal, symbolic and graphic approach according to Duval's Theory, based on a computational tool, under the following process:

- ✓ Analysis of the student's graduation profile (Select and apply mathematical tools for modeling, design and development of computational technology).
- ✓ Characterization of the subject (Contribute to develop a logical-mathematical thought to the engineer's profile and provide the basic tools to enter the study of calculation and its application).
- ✓ Didactic intention (Intuitively approach the derivative, obtaining the slope of the line tangent to a curve).

- ✓ Derivative competence (Use the formula and definition of the derivative for the analysis of functions and calculation of derivatives).
- ✓ Learning activity of the derivative competence (Show that the value of the slope of the tangent to a curve at one point can be obtained by calculating the derivative of the function corresponding to the curve at that point).

Considering the above as the didactic intention and activities of the derivative competence established in the study program of the subject of Differential Calculus (Instituto Tecnológico de Comitán, 2018), the following activities are implemented to encourage student learning.

Activity 1. Formula and definition of the derivative of a function at a point.
First, it provides students with both the formula and the definition of the derivative.

Formula: $$f(x)' = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$$

Definition: *It is the slope of a tangent line to a curve at a given point.*

This activity has the purpose that the students are related to the basic concepts and give way to the following activity to identify by simple observation the mathematical concepts involved.

Activity 2. Identification of the necessary previous knowledge.

Based on the formula and definition of the derivative, it is important that the necessary prior knowledge be identified, to continue with the explanation of the mathematical concepts involved, such as: secant line, tangent line, slope of a line, differentials and limit of a function. It can be the situation that students fail to identify all the concepts involved, such as the secant line. However, when mentioning the tangent line, it is important the facilitator's intervention to justify his consideration in the explanation and analysis.

Activity 3. Explanation of the necessary previous knowledge (written or symbolic form and with the use of a computational tool).

This activity consists in explaining each of the mathematical concepts involved, both its definition (verbal representation), its written or symbolic representation (Figures 1, 3, 5 and 7) and graphic through the use of a computational tool (Figures 2, 4 , 6, 8, 9, 10 and 11). This part of the activity is considered very important so that the student gradually builds the mathematical concept and can be used in contexts of his academic interest, such as Salinas, Pulido and Alanís (2016) mention it in his book on innovation in Teaching-learning calculus for engineering careers.

Definition of the secant line (Verbal representation): *Let P_1 be a point on the curve and let P_2 be a moving point close to P_1 on that curve, the line that passes through P_1 and P_2 is called the secant line* (Purcell, Varberg & Rigdon, 2007, p. 93).

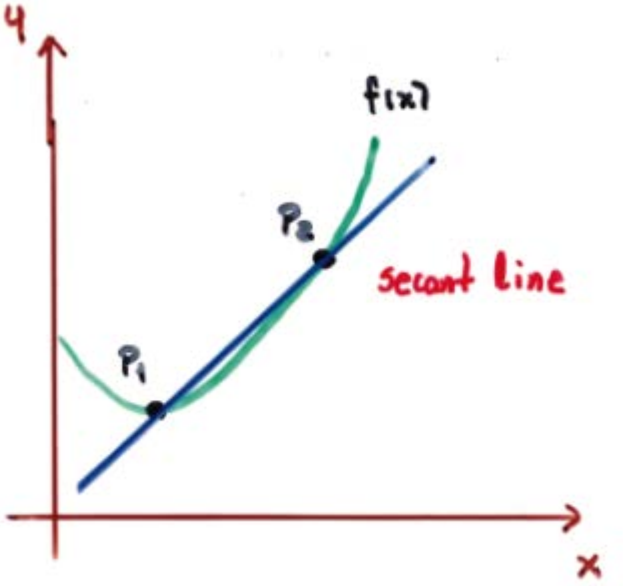

Figure 1. Written representation of the secant line.

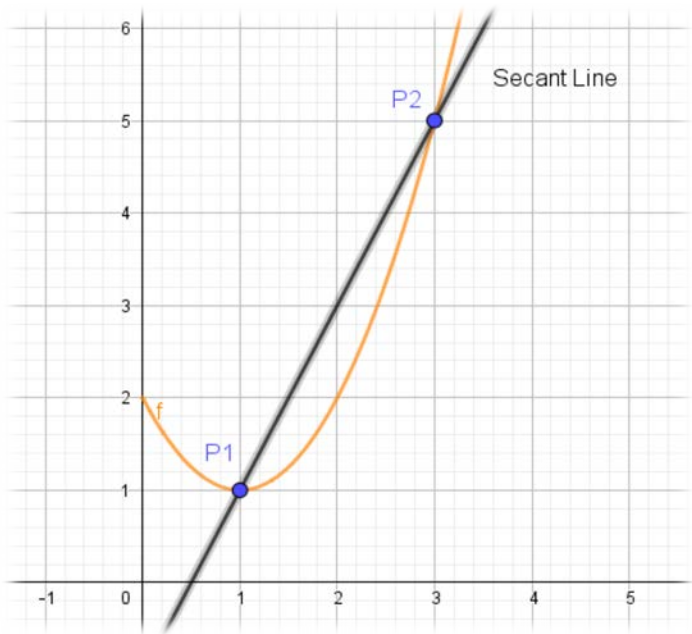

Figure 2. Graphical representation of the secant line through the use of the computational tool.

Medina, A., Flores, M. G., & Rosas, A. (2019). A didactic sequence for the geometric interpretation of the derivative. In A. Rosas (Ed.), *Research Reports in Mathematics Education: the classroom* (pp. 83-100). Miami, Florida: L.D. Books.

Definition of the tangent line (Verbal representation): *The tangent line in P_1 is the boundary position (if it exists) of the secant line when P_2 moves towards P_1 along the curve* (Purcell, Varberg y Rigdon, 2007, p. 93).

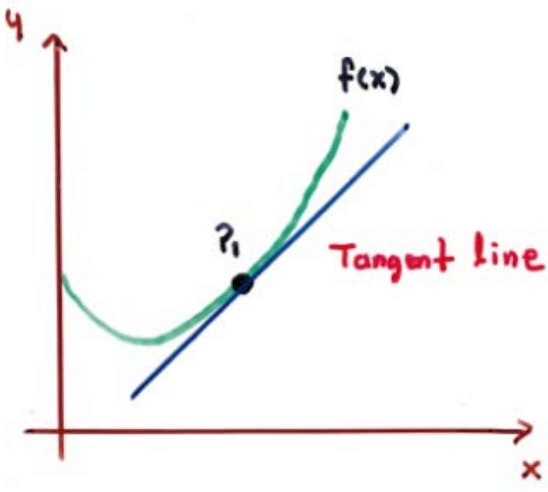

Figure 3. Written representation of the tangent line.

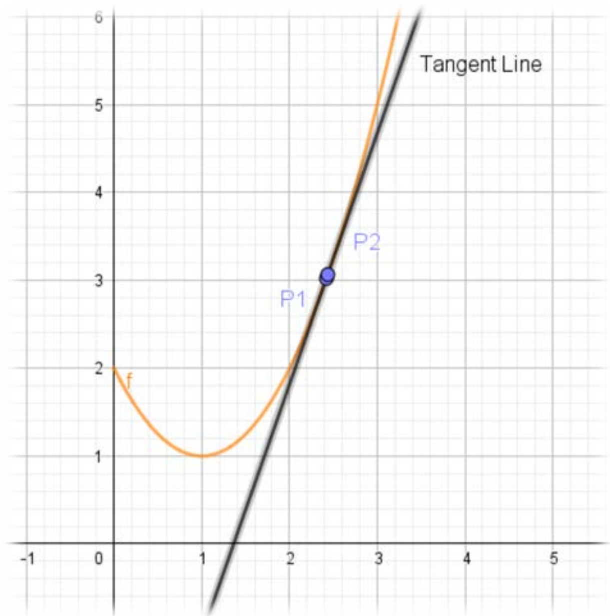

Figure 4. Graphical representation of the tangent line through the use of the computational tool.

Definition of the slope of a line (Verbal representation): It is a measure of the inclination of the line. It is the reason between the change in y, Δ_y, *and the change in x*, Δ_x (Stewart, 2010, p. A10).

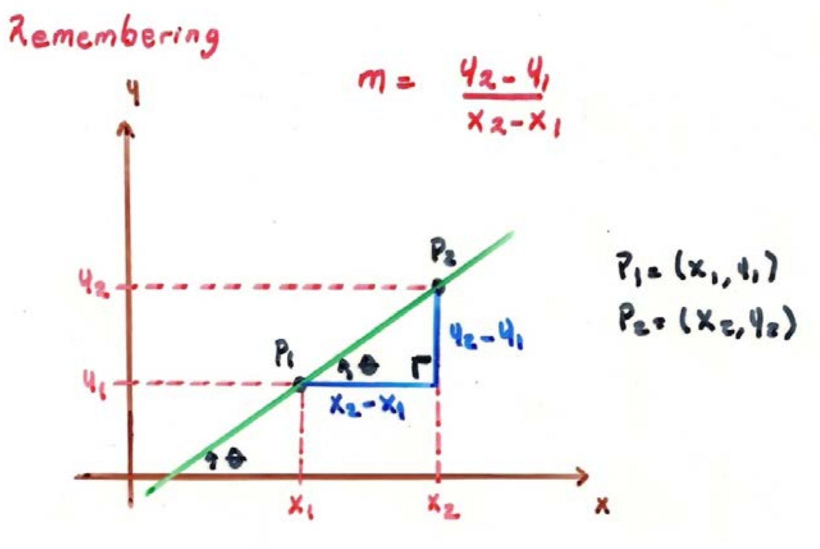

Figure 5. Written representation of the slope of a line

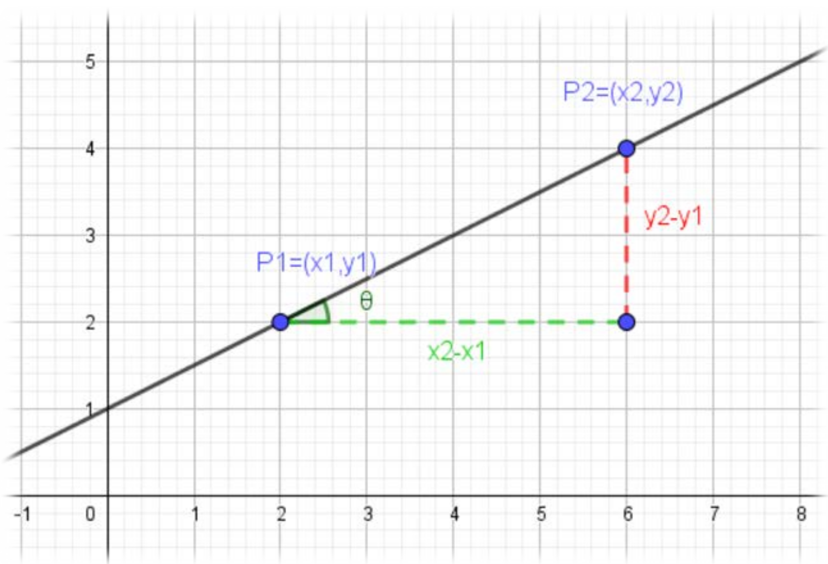

Figure 6. Graphical representation of the slope of a line through the use of the computational tool

Differential definition (Verbal representation): If the value of a variable x changes from x_1 *to* x_2*, then* $x_2 - x_1$*, the change of x, is called an increase of x and is usually denoted by* Δ_x*, in the same way for the variable y* (Purcell, Varberg y Rigdon, 2007, p.143).

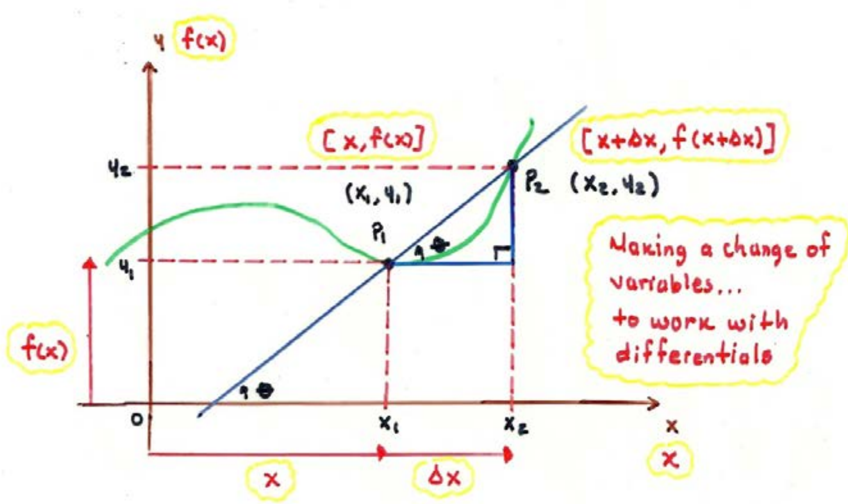

Figure 7. Written representation of the differentials

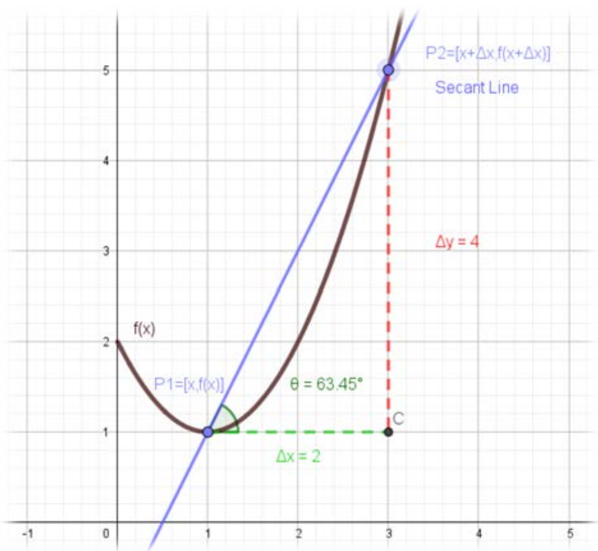

Figure 8. Graphical representation of the differentials ($\Delta_x = 2$).

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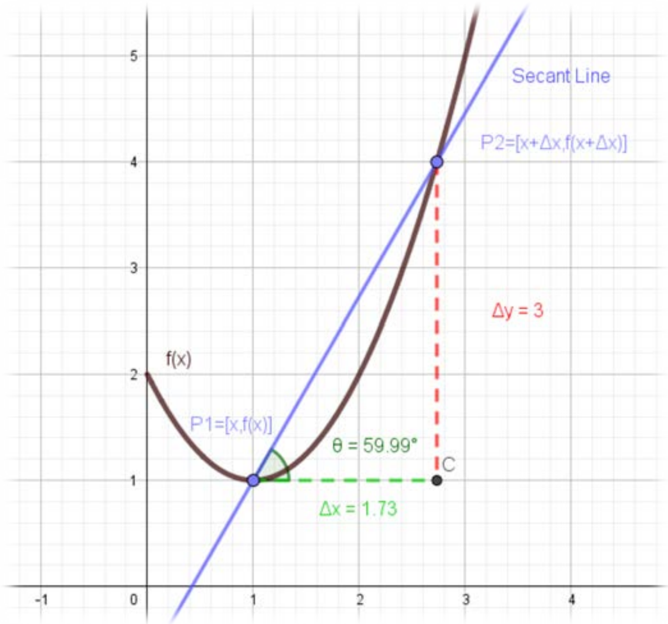

Figure 9. Graphical representation of the differentials ($\Delta_x = 1.73$).

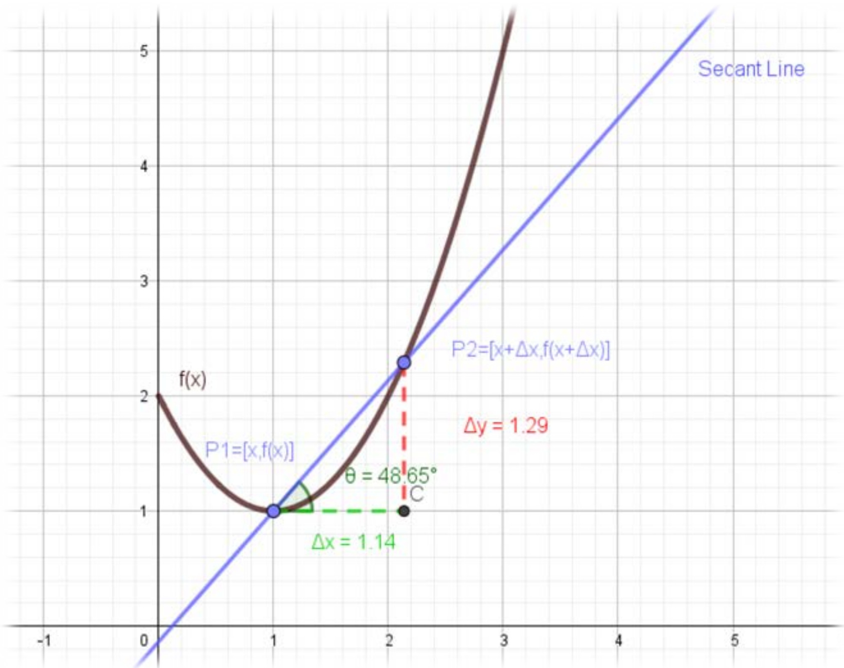

Figure 10. Graphical representation of the differentials ($\Delta_x = 1.14$).

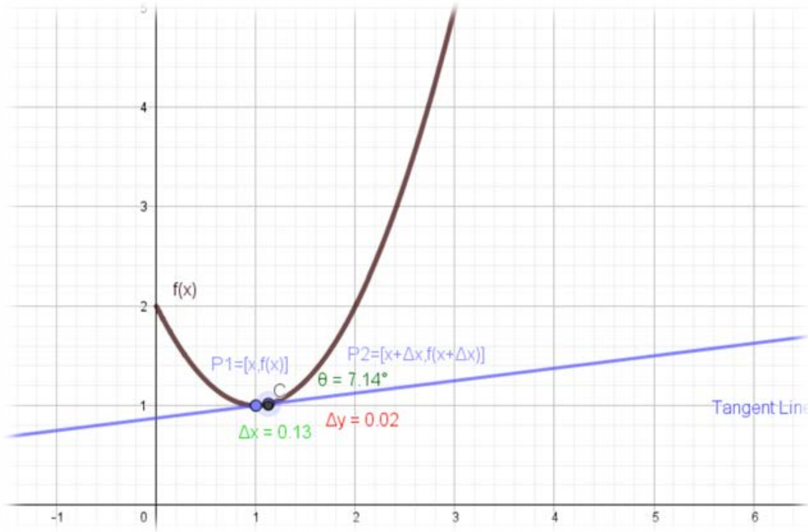

Figure 11. Graphical representation of the differentials ($\Delta_x \rightarrow 0$).

Figures 8, 9, 10 and 11 show the process of the limit, *as point P_2 approaches point P_1 along the parabola, the corresponding secant lines rotate around P_1 and approximates the tangent line.*

Activity 4. Geometric Interpretation of the Derivative (written form and with the use of a computational tool).

As a closing activity of the didactic sequence, the idea is to show students both the formula and the definition of the derivative of a function at one point, for their analysis and reflection of the mathematical concept, which allows them to recognize the notions and procedures involved Analyzing the expression from right to left, it can be observed, as from a slope (secant line), the limit is applied when it tends to zero obtaining a tangent line, which geometrically means the derivative of a function at a given point.

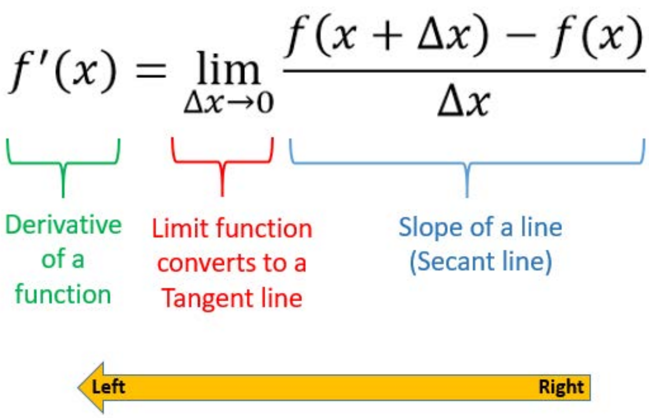

Figure 12. Formula of the derivative of a function at a point.

Final Remarks

The didactic sequence of the formula and definition of the derivative of a function at one point or another mathematical concept benefits us to organize the content that will be taught in the classroom and the methodology used allows us to know the previous knowledge required for the student understand the geometric interpretation of the derivative that will help us better understand both the formula and the definition of the derivative, such as: the secant line, tangent line, slope of a line, differentials, limit of a function, among others, also make different representations such as verbal, written (symbolic) and the use of a computational tool (graph), contributes to the same mathematical concept is represented in different ways for a better understanding, in accordance with the previously reported (Chang et al, 2015).

A preliminary way we can comment that the six activities of the sequence have already been applied, the research population consisted of 30 students of the first semester of the Computer Systems Engineering degree of the Technological Institute of Comitán, during the semester Aug-Dec-2018.

The methodological approach was carried out qualitatively and we can mention that we managed to develop a didactic sequence for the understanding of the formula and definition of the derivative.

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The teaching of statistics to future nurses: a didactic proposal

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Abstract

This article describes the design and implementation of a didactic proposal for teaching and learning statistics in the training of nurses. The proposal is based on a thorough analysis of research articles on specific, contextualized problems in the field of nursing that involve the statistical concepts studied in this discipline in the nursing program of a Colombian university. Both the analysis of the articles and the research activity are framed in terms of the research cycle of the statistical thought model developed by Wild and Pfannkuch (1999). The preliminary results of the proposal give an account of its potential to foment, in students, the development of a greater understanding of the statistical concepts required to evaluate situations or problems in the field of nursing.

Keywords: statistical thinking, statistical training for nurses, introductory statistics courses.

1. Introduction

According to Garfield, Hogg, Schau and Whittinghill (2002), introductory statistics courses are a focus of attention in current recommendations for changes in statistical education. This research emerges from problems related to these introductory courses, especially the teaching of statistics in the training of future nurses. Gatusso and Ottaviani (2011) report that

Feo, I., & Gómez-Blancarte, A. (2019). The teaching of statistics to future nurses: a didactic proposal. In A. Rosas (Ed.), *Research Reports in Mathematics Education: the classroom* (pp. 101-115). Miami, FL: L.D. Books.

the teaching of statistics is usually performed through an approach that is more procedural than conceptual; that is, from a mathematical perspective that places greater emphasis on methods than on understanding statistical ideas. Moreover, statistics is usually taught in a decontextualized manner, with little effort made to deepen the interpretation of statistical results in relation to real contexts. This way of approaching the teaching of statistics has led students in various disciplines, like the nursing students reported here, to question the importance of the subject of statistics for their training. Given this situation, there is a need to seek strategies to address the study of statistics so that nursing students develop statistical ideas that are more conceptual than procedural in nature, and so come to recognize the usefulness of this discipline for their field of training.

This article presents advances in research related to the implementation of a didactic strategy that consists in analyzing research articles from the field of nursing. The aim of this approach is to help students develop their understanding of elements of statistical thinking. The analysis of research articles has been used as an evaluative resource to foment the interpretation of statistical results applied in the context of medicine (e.g. McNiece, 2010). The present proposal considers that this analysis can be used as both a teaching strategy and a method of evaluation.

2. Conceptual framework

Promoting the development of statistical thinking is a current recommendation posited to reorient the approach to teaching statistics (Ben-Zvi & Garfield, 2004; Garfield & Ben-Zvi, 2008; Batanero, Burril & Reading, 2011; American Statistical Association [ASA], 2016). In the present study, therefore, the statistical thought model was used as a reference framework for analyzing research articles. According to Wild and Pfannkuch (1999), when a researcher works on a real statistical problem, four interrelated dimensions come into play continuously and simultaneously (Figure 1). These four dimensions make up a model of statistical thinking that consists of: a research cycle (dimension 1), types of statistical thinking (dimension 2), an interrogative cycle (dimension 3), and provisions (dimension 4).

The fundamental types of statistical thinking are related to cognitive activity, while the elements of the research cycle are associated with activities that are more operational in nature. The elements of the interrogative cycle, meanwhile, seek to question the application of certain statistical procedures and concepts required to solve a problem. For this, it is necessary to look for sources, data and information that will lead to a clear interpretation of the phenomenon under study and its results, and to subject the research process to a level of criticism that allows the researcher to stop along the way and judge the processes while they are being carried out, along with the seriousness of the results obtained. Provisions come into play in the fourth, and final, dimension, which refers to personal qualities that may affect an individual's commitment to research and the analysis and interpretation of results.

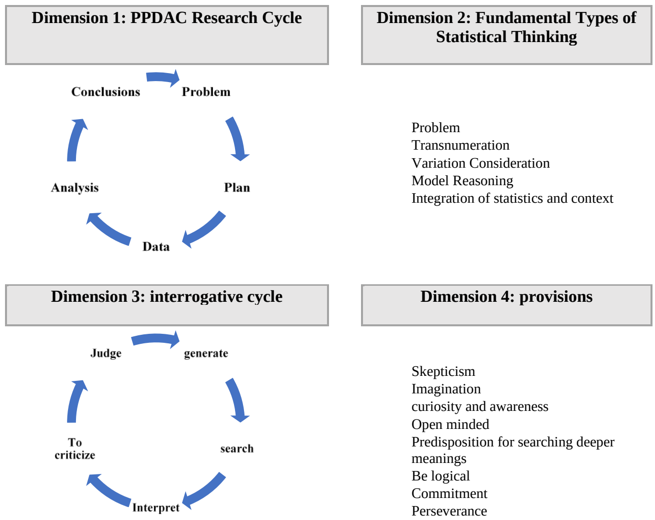

Figure 1. Model of statistical thinking. Adapted from Wild and Pfannkuch (1999).

These four dimensions represent elements of statistical thinking that are neither hierarchical nor linear; indeed, several may concur in the same phase of research. In this article, the PPDAC research cycle was applied to the analysis of research articles following the sequence: Problem-Plan-Data-Analysis--Conclusions (Figure 2). These phases make it possible to structure projects from the approach to a research issue or statistical problem (hereinafter, the Problem) to the conclusion, which entails integrating knowledge of the context. The issue examined herein derives from a real-world problem for which statistics can provide a solution of, or a judgment regarding, the situation. This real-world problem can be framed in any field of knowledge, but in this case we chose nursing.

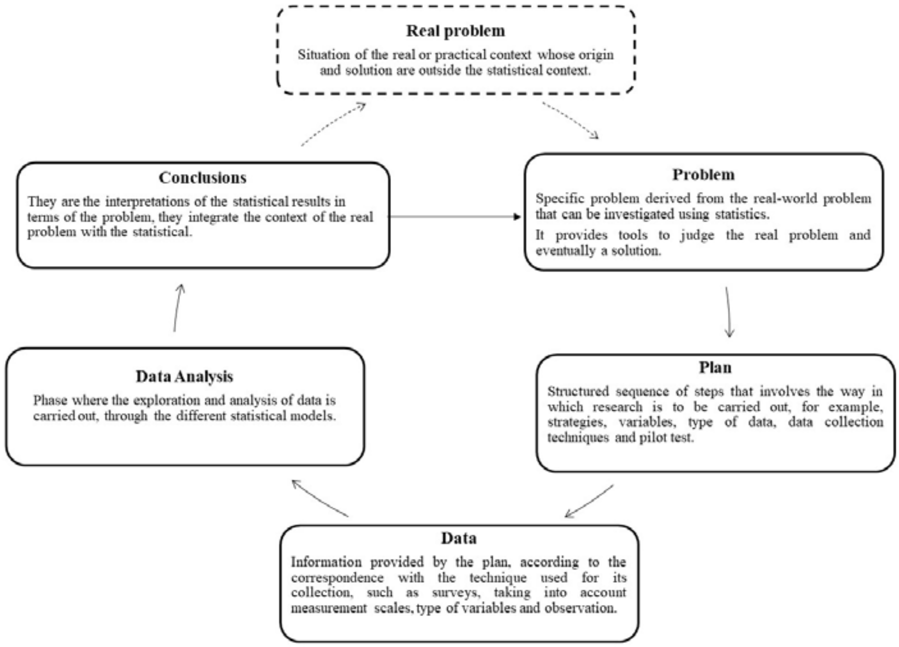

Figure 2. PPDAC research cycle. Adapted from Wild and Pfannkuch (1999)

3. Design of the didactic proposal

The didactic proposal to analyze research articles in the field of nursing was designed so that it could be integrated into the requirements of the Study Program –both content and evaluation– of a course on Statistics for the nursing career at the University of Applied and Environmental Sciences (UDCA). Regarding content, the Program suggests studying three principle statistical topics: 1) Data exploration; 2) Introduction to probability; and 3) Inferences about variables. The selection of the articles to be analyzed was carried out according to three main criteria: (a) research articles from nursing research journals that (b) address some of the contents of the subject of statistics and that

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(c) are written in Spanish. To carry out this proposal, a set of educational activities was designed and organized in three phases:

Phase 0: Introduction to the analysis of research articles. The objective of this phase was to introduce students to the analysis of articles based on Wild and Pfannkuch's statistical thought model (1999), specifically dimension 1: the PPDAC research cycle.

Phase 1: Analysis of research articles. In this phase students were instructed to analyze two research articles in order to identify the five phases of the PPDAC cycle, and then discuss the statistical concepts and procedures entailed in each phase.

Phase 2: Research project. In this phase, the students carried out, and then presented, a research project on a topic related to nursing. The objective of this phase was for students to conduct their own research in order to practice applying elements of statistical thinking.

3.1. Participants

The participants were nursing students from the U.D.C.A. in Bogotá D.C., Colombia, enrolled in a course on Statistics in the second semester of 2017. The group consisted of 20 students (4 men, 16 women) aged 18-21.

3.2. Implementation of the didactic proposal

Each phase was integrated into the activities programmed in the students' course on Statistics. Together they took up 6 weeks of that 16-week course (see Table 1).

Table 1.
Timeline for implementing teaching activities

Date	Week(s)	Activities	Time
Phase 0: Introduction to the analysis of research articles			
August 23rd 2017	4	Reading of article 1 by Aya and Suárez (2015).	3 extra-class hours
August 30th, 2017	5	<u>First moment</u>: PowerPoint presentation of article 1 by the teacher. Introduction to the PPDAC cycle.	4 extra-class hours
		Reading of article 2 by Aguilar and Flórez (2016). Training of 7 teams of 3 students each.	3 extra-class hours
		Socialization and parameters for the final research activity (corresponding to week 16, Phase 2).	
September 4th and 6th	6	<u>Second moment</u>: presentation of the analysis of article 2 by the 7 teams.	4 class hours
	Phase 1: Analysis of research articles		
October 23rd and 30th, 2017	13, 14	Reading of two research articles (#3, 4). Article 3 by Pérez, López, Benítez and Sandoval (2011); article 4 by Arias, Palomino and Agudelo (2014). Here, 4 teams of 5 students each were formed.	3 extra-class hours
		Analysis and presentation of articles 3 and 4 by students.	4 extra-class hours
	Phase 2: Research project		
November 20th and 22nd	16	Submission and presentation of the research activity.	4 extra-class hours

Note. Elaborated by the author

Each phase of the didactic proposal was implemented in class as a complement to the contents of the study plan for the Statistics course.

3.2.1. Example of the analysis of research articles

Examples of the analyses of article 2 according to the PPDAC research cycle are shown in Figure 3, where teams were instructed to organize the reading of the articles cited in Table 1.

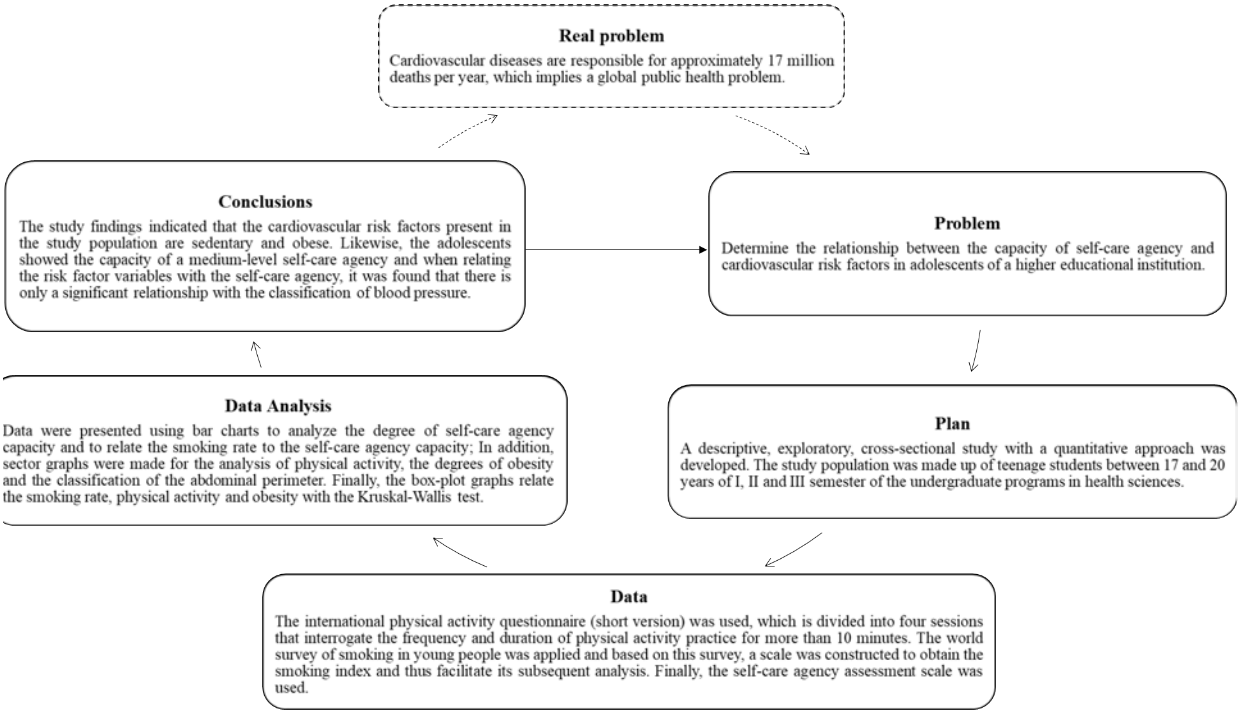

Figure 3. Analysis of article 2, according to the PPDAC cycle.

3.3. Data analysis

For data collection and analysis, videos of the students' exhibitions were taken in phases 1 and 2 and then transcribed in order to search for, and capture, the situations that demonstrated how students were beginning to understand and apply elements of the statistical thought model.

4. Results

This article reports only the analysis of the second moment of phase 1 (6^{th} week). The transcripts presented below are excerpts of the explanations offered by the students themselves according to their analyses of article 2 (see Figure 3). These transcripts were chosen to illustrate students' recognition of the elements of the PPDAC cycle.

4.1. Recognition of the real problem and the Problem

The following excerpt relates how a real problem was differentiated from a Problem. This first transcript presents group number 1 (team 1) and one of its members (Expo-1), in dialogue with the professor and other students who were not presenting at that time. They are identified accordingly: Professor, Stu-audience.

Expo-1-team 1:	*[...] the statistical problem is: the self-care agency and cardiovascular risk factors in adolescents at an institution of higher education in Barranquilla [...]*
Professor	*[...] the article tells us about self-care and cardiovascular risk factors, [...] where the research question or statistical problem arises [...]*
Expo-1-team 1:	*[...] the real context [...] of the factors that can lead us to suffer cardiovascular disease*
Student_Grooup	*Cardiovascular diseases*

The student identifies the Problem (*i.e.* to determine the relation between the capacity of a self-care agency and cardiovascular risk factors). The student audience intervenes and, in one voice, identify the real problem (cardiovascular diseases) from which the research emerged.

4.2. Plan Recognition

In the intervention by team two (group 2), one member (Expo-2) explained how the plan was carried out in the article in order to respond to the Problem.

Expo-2-team 2 *[...] having determined the problems [real and statistical], the plan tells us to first look at what kind of study the people who did this research conducted. This was a descriptive study. [...] They used quantitative variables [...], they were teenage students between 17 and 20 years old [...]; the sample had a total of 133 students [...] the criterion for choosing this sample was that they did not have cardiovascular pathologies and that the women were not pregnant.* [...]

The student-presenter expressed that having determined the question, they needed to devise a way to collect the data, including identifying and type(s) of data to be gathered and the method for doing so. The student also considered the characteristics of the sample and the method to be used to select it, both of which were clearly identified by the team in their analysis of the article.

Each team's analysis, together with their presentations, reflect how they were able to identify statistical concepts that are usually studied in courses on data exploration (e.g. variables, data, samples, population). This allows us to assume that by analyzing these research articles the students endowed these topics with greater meaning, since they are able to relate them to a context that forms part of their training.

4.3. Data Recognition

In this phase of the Data step, the student-presenter (Expo-3) explained the technique employed to collect the data (World Smoking Survey) and identified the types of variables.

The use of concepts (scale) and terms (physical activity, smoking) from the field of nursing in this presentation is striking.

Expo-3-team 3	*[...] both variable classes (qualitative and quantitative) were handled for physical activity and smoking, because in physical activity, the part of the items also referred to how many hours a day. [...]the World Tobacco Survey refers to the qualitative and quantitative parts because they ask if they smoked, [...] how many cigarettes [...] why they had begun to smoke, and if they had quit. Then they began to ask sporadic questions, so that survey handled both types of variables, qualitative and quantitative [...]*
Professor:	*[...] the researchers who applied the World Survey on Smoking in young people used a strategy to obtain data. What did they do?*
Group:	*A scale [chart that offers a way to keep adjusted accounts].*
Student-Group	*I'm not sure, but I think it's the activity of organizing the data [...] to get a scale of smoking [...]*

4.4. Data Analysis Recognition

In the Analysis phase, students were asked to identify the processes of data transformation (transnumeration) applied to organize, classify and develop models in the form of graphical representations. In the following extract, a member of group 3 describes how the article analyzed and processed the data.

Expo-3-team 3	*[...] the data were processed in an Excel data-sorting table and the Kruskal Wallis test was used to check significance levels [...] also, they say the box plot was used [...]*
Professor:	*[...] how is the Kruskal Wallis test handled to understand data analysis?*
Expo-3-team 3	*[...] well, according to my question [...] it's a non-parametric method to test whether a group of data comes from the same population [...] It's a relation of the median that's what you're going to analyze [...]*

We note in this intervention that the student exhibitor is clear about how the data were analyzed and mentions topics that had not yet been studied in their course, such as hypothesis tests (Kruskal Wallis). In this sense, the analysis of the article favored a more holistic teaching of the contents, since the students did not need to wait until certain elements were presented in class following the sequence indicated in the Study Plan.

4.5. Recognition of Conclusions

In the course of the data analysis generated by the article, the students were participatory and critical, as shown by their comments in the interpretation and analysis of graph 5 from the article (see Figure 4) on the risk of cardiovascular diseases.

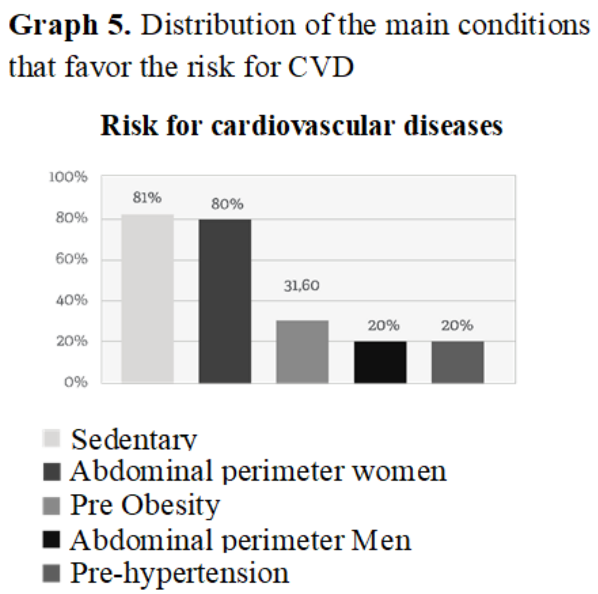

Figure 4. Risk of Cardiovascular Diseases. Aguilar and Flórez (2016)

Student-Group	*Well, I want to say that most respondents have sedentary lifestyles. This implies that most of them have a normal state of blood pressure; that is to say, the consequences of a sedentary lifestyle are not felt only at this age but at older ages as well.*
Group	*No, sedentary lifestyle can lead to hypertension.*
Student-Group	*Therefore, if the majority are sedentary, this means that not being sedentary at my age, I have hypertension even before I get older.*
Group	*In the future you would have.*
Student-Group	*Because that way most of you here would suffer hypertension.*

Reaching this conclusion meant that each result had to be interpreted. This leads us right back to the beginning of the project and the aim of responding to the initial question; that is, the Problem. In this case, students showed how they related the data to the context, and developed the ability to criticize, interpret and judge the results. In addition, they began to look for deeper meanings and to open their minds to receive new knowledge.

5. Conclusion

The use of the PPDAC cycle helped favor the recognition of the stages of research shown in the article that, in turn, stimulated discussion of the statistical concepts and processes involved in each stage. The way in which the students treated each element of the cycle showed how they had come to understand both the statistical concepts and processes studied in their course on statistics, as well as several not included in that study plan. In this sense, analyzing the article proposed at the outset revealed students' potential to acquire an understanding not only of the contents studied, but also of other elements not yet seen, in order to comprehend and evaluate the research problem and the results presented in the article.

The preliminary results presented in this article indicate how students can develop statistical thinking that will allow them to understand statistical ideas, from the integration of the context with the statistical approach, in order to achieve a better understanding of the statistical concepts and procedures normally studied in standard statistics courses. In addition, the strategy suggests that students developed elements, such as curiosity and predisposition, to deepen the meanings of the statistical results.

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